

## Pre-calculus NC Common Exam Project

### Part A: Work out problems correctly- Completion Grade Due May 18, 2015

Each group will be assigned several problems to complete. You are to work out your assigned problems (as a group) and turn them in with all the work by the due date above. I then will provide feedback and corrections so that when you present to the class your answers are correct. **PLEASE BE SURE TO SHOW ALL YOUR WORK AND YOUR WORK MUST SUPPORT YOUR ANSWER!**

### Part B: Presentation and explanation to class- Quiz Grade Presentations begin May 26, 2015

- You will present your questions at the very beginning of class, and you will only be given 15 minutes MAX.
- Explain answers in full detail and be able to answer any question from students. IF STUDENTS DO NOT HAVE QUESTIONS, BE PREPARED TO ASK THEM QUESTIONS TO CHECK THEIR UNDERSTANDING.
- During your explanation, provide any formulas/ information/notes necessary for students to remember concerning this question/topic.
- Explain what difficulties you had or what you had to research concerning the question/topic.
- Each individual will write a short paragraph reflecting on your "teaching" experience. Make sure to answer the following questions in complete sentences: What challenges did you face concerning the material? What challenges did you face with the class? Did you learn anything? Did you find this exercise helpful? What was your role in the group? What work did you do?
- Your peers and I will grade your presentation according to the rubrics below.

**Teacher Rubric**

Quiz Grade	Points Possible	Points Received
Time- Students took no more than 15 minutes to complete their presentation	15	
Explanation to class	25	
Things to remember: Formulas/notes/tips to solve	10	
Reflection (Due day after presentation)	15	
Average of Student Rubrics	35	
<b>Total</b>	<b>100</b>	

**Student Rubric**

Quiz Grade	Points Possible	Points Received
Explanation of answers	10	
Answered questions from class	10	
Every member participated	5	
Overall Effort	10	
<b>Total</b>	<b>35</b>	

### Part C: Binder with all solutions and notes- Test Grade Due May 29, 2015

Once the last group has presented, you will have seen all the answers of the Final Exam Review. At this point, you should insure that your solutions are correct, the work is correct, add any tips or suggestions shown by presenters, and your notebook is ready to be turned in on the due date. The following rubric is how your completed final exam will be graded.

Test Grade	Points Possible	Points Received
Neatness (a notebook is required; page protectors are NOT REQUIRED)	16	
Solutions- not only do you have the correct answer, but the work and reasoning behind it.	56	
Things to remember/notes: For each question you have study tips or formulas to remember.	28	
<b>Total</b>	<b>100</b>	

## Math Formula Sheet for AFM and Pre-Calculus

### Arithmetic Sequence and Series

$$a_n = a_1 + (n - 1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

### Geometric Sequence and Series

$$a_n = a_1 \cdot r^{(n-1)}$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}, \text{ where } r \neq 1$$

$$S = \frac{a_1}{1 - r}, \text{ where } |r| < 1$$

### Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

### Conic Sections

#### Parabola

#### Focal Length

$$|a| = \frac{1}{4c}$$

#### Ellipse

#### Pythagorean Relationship

$$c^2 = a^2 - b^2$$

#### Hyperbola with Center $(h, k)$

#### Pythagorean Relationship

$$c^2 = a^2 + b^2$$

#### Foci

$$(h \pm c, k) \text{ or } (h, k \pm c)$$



1 What are the **approximate** rectangular coordinates for the point with polar coordinates  $(5, 30^\circ)$ ?

- A (2.5, 2.89)
- B (2.5, 4.33)
- C (2.89, 4.33)
- D (4.33, 2.5)

2 A sequence is shown below.

6, 12, 20, 30, 42, 56, ...

Which is the recursive formula for this sequence?

- A  $t_n = n + 2(t_{n-1} + 1)$
- B  $t_n = (t_{n-1} + 1)(n - 2)$
- C  $t_n = 2(t_{n-1} + 2) - (n + 2)$
- D  $t_n = t_{n-1} + 2(n + 1)$



3 A quadratic function,  $f$ , has zeros  $P$  and  $Q$ , such that  $P + Q = 5$  and  $\frac{1}{P} + \frac{1}{Q} = 8$ . Which choice describes  $f$ ?

- A  $f(x) = 8x^2 - 40x + 5$
- B  $f(x) = 8x^2 - 40x - 5$
- C  $f(x) = 2x^2 - 10x + 5$
- D  $f(x) = 2x^2 - 10x - 5$

4 Lucy invested \$6,000 into an account that earns 5% interest compounded continuously. **Approximately** how long will it take for Lucy's investment to be valued at \$25,000?

- A 52.7 years
- B 46.9 years
- C 24.5 years
- D 23.8 years

5 A lamppost is located 18 feet from a building. The angle of elevation from the base of the lamppost to the top of the building is  $32.3^\circ$ . **Approximately** how tall is the building?

- A 223 feet
- B 264 feet
- C 510 feet
- D 661 feet

RELEASED

RELEASED



6 Two functions are shown below.

$$T(x) = -x$$

$$P(x) = 10x + 2$$

What is the value of  $P(T(3)) - T(P(3))$ ?

- A 8
- B 4
- C 0
- D -4

7 A piecewise function is shown below.

$$f(x) = \begin{cases} cx - 1 & \text{if } x \leq 2 \\ cx + 2 & \text{if } x > 2 \end{cases}$$

For what value of  $c$  does  $\lim_{x \rightarrow 2} f(x)$  exist?

- A -2
- B -1
- C 1
- D 4

ANSWERS — RELEASED FORM

Go to the next page



8 What are the polar coordinates of  $(4, 9)$ ?

- A  $(\sqrt{97}, 66^\circ)$
- B  $(\sqrt{97}, 114^\circ)$
- C  $(\sqrt{13}, 66^\circ)$
- D  $(\sqrt{13}, 114^\circ)$

9 A sequence is shown below.

$$1, 3, 3^2, 3^3, \dots$$

How many terms of the sequence must be added together for the sum to equal 3,280?

- A 6
- B 7
- C 8
- D 9

ANSWERS — RELEASED FORM

Go to the next page



10 The first term of an infinite geometric sequence is 2. The sum of the sequence is 6. What is the common ratio of the sequence?

- A  $\frac{1}{3}$
- B  $\frac{2}{3}$
- C  $\frac{3}{3}$
- D  $\frac{4}{3}$

11 Which is true of the series shown below?

$$\pi + \frac{3\pi}{4} + \frac{9\pi}{16} + \frac{27\pi}{64} + \dots$$

- A The series diverges.
- B The series converges to  $\frac{3\pi}{2}$ .
- C The series converges to  $\frac{9\pi}{3}$ .
- D The series converges to  $4\pi$ .

12 Karen recursively generated a sequence of five positive integers by starting with a positive integer,  $a_1$ , and then applying the recursive formula  $a_n = a_{n-1} + 3n - 1$  to generate  $a_n$  for  $n = 2, 3, 4$ , and 5.

- If the value of  $a_5$  was 407, what was the value of Karen's starting term,  $a_1$ ?
- A 366
  - B 367
  - C 368
  - D 369

13 What is the distance between y-intercepts of the graph of  $x + 8 = 2(y + 3)^2$ ?

- A 4
- B 6
- C 11
- D 15

14 Which is a solution set to  $\frac{3x}{x-1} = \frac{x+2}{x-1}$ ?

- A  $\{-1\}$
- B  $\{2\}$
- C  $\{2, 1\}$
- D  $\{2, -1\}$



15 What is the range of the inverse of  $y = \tan x$ ?

- A  $-\frac{\pi}{2} < y < \frac{\pi}{2}$
- B  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
- C  $0 < y < \pi$
- D  $0 \leq y \leq \pi$

16 James is standing 10 meters away from Samantha.  
 • A bird is located in the sky at a point between where James and Samantha are standing.  
 • James is looking up at the bird at an angle of elevation of  $74^\circ$ .  
 • Samantha is looking up at the bird at an angle of elevation of  $47^\circ$ .

Approximately how far is the bird from Samantha?

- A 7.6 meters
- B 8.5 meters
- C 11.2 meters
- D 13.1 meters



17 What is the inverse function of  $f(x) = \log_5(2x - 1)$ ?

- A  $f^{-1}(x) = 5^x - 1$
- B  $f^{-1}(x) = \frac{5^x + 1}{2}$
- C  $f^{-1}(x) = \log_5(5x - 1)$
- D  $f^{-1}(x) = \log_5 \frac{5x + 1}{2}$

18 What is the value of the limit shown below?

$$\lim_{n \rightarrow \infty} \left( \frac{3^n - 1}{-3^n} \right)$$

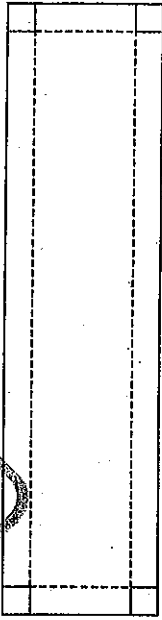
- A  $\frac{1}{3}$
- B  $\frac{2}{3}$
- C 1
- D  $+\infty$

19 What type of conic section is represented by  $r = \frac{8}{16 + 125 \sin \theta}$ ?

- A circle
- B ellipse
- C hyperbola
- D parabola



- 20 James had a rectangular piece of cardboard that was four times as long as it was wide. He wanted to use the cardboard to make a box with no lid. To do this, he first cut a 3-by-3-inch square out of each of the four corners of the piece of cardboard, as shown in the picture below.



Then James folded the cardboard along the four indicated lines shown in the picture. This created an open box with a volume of 836 cubic inches.

What was the width of the sheet of cardboard that James started with?

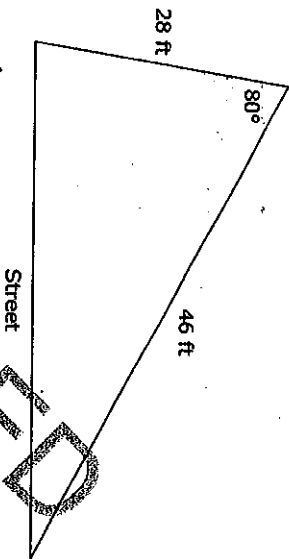
- A 10.5 inches
- B 9.5 inches
- C 8.5 inches
- D 7.5 inches

- 21 Which expression is equivalent to  $(\sec \theta) \left( \frac{\sin \theta}{\tan \theta} \right)$ ?

- A  $\cos^2 \theta - \sin^2 \theta$
- B  $\sin^2 \theta - \cos^2 \theta$
- C  $\cot^2 \theta - \csc^2 \theta$
- D  $\csc^2 \theta - \cot^2 \theta$



- 22 Suppose that for each foot of land along the street, the annual tax is \$25 per foot. The diagram below shows a plot of land.



About how much is the annual tax for the plot?

- A \$1,238
- B \$1,293
- C \$1,321
- D \$1,411

- 23 The function  $C(x) = \frac{2.50x + 1.00}{x}$  models the cost per item for a company to produce  $x$  items after the first item is made. What is the inverse function of  $C(x)$ ?

- A  $C^{-1}(x) = \frac{1.00}{x - 2.50}$
- B  $C^{-1}(x) = \frac{x - 2.50}{1.00}$
- C  $C^{-1}(x) = \frac{x - 1.00}{2.50}$
- D  $C^{-1}(x) = \frac{2.50}{x - 1.00}$



- 24 A computer rental company charges \$50 to rent a computer for one week. The table below shows the daily late fees the company charges if a computer is returned late.

Days Late	Daily Late Fee
days 1 through 10	\$5
days 11 through 20	\$8
days 21 through 30	\$10

What would be the total cost of renting a computer for one week and returning it 15 days late?

- A \$120
- B \$125
- C \$140
- D \$170

- 25 From a point 100 feet from the base of a building, Angie looks up at a  $40^\circ$  angle to the top of a building. She walks 20 feet closer to the building. At *approximately* what angle must Angie now look up to see the top of the building?

- A  $32^\circ$
- B  $46^\circ$
- C  $60^\circ$
- D  $77^\circ$



- 26 What transformations have occurred to create the function  $f(x) = 3x^2 - 4$  from the function  $g(x) = x^2$ ?
- A The graph of the function has been stretched horizontally and shifted up four units.
  - B The graph of the function has been stretched vertically and shifted up four units.
  - C The graph of the function has been stretched horizontally and shifted down four units.
  - D The graph of the function has been stretched vertically and shifted down four units.
- 27 An object is launched straight upward from ground level with an initial velocity of 50.0 feet per second. The height,  $h$  (in feet above ground level), of the object  $t$  seconds after the launch is given by the function  $h(t) = -16t^2 + 50t$ . At *approximately* what value of  $t$  will the object have a height of 28.0 feet and be traveling downward?
- A 2.39 seconds
  - B 1.84 seconds
  - C 1.56 seconds
  - D 0.73 seconds
- 28 What is the range of the function  $f(x) = -5 - 2(x + 3)^2$ ?
- A  $[-5, \infty)$
  - B  $(-\infty, 5]$
  - C  $(-\infty, -5]$
  - D  $(-\infty, \infty)$





- 24 A wind that is blowing from the northwest toward the southeast can be represented by a vector. The vector has an eastward component and a southward component. If the eastward component has a magnitude of 5.00 miles per hour and the southward component has a magnitude of 15.00 miles per hour, in what direction is the wind blowing?

- A The wind is blowing in the direction  $71.6^\circ$  east of south.
- B The wind is blowing in the direction  $67.5^\circ$  east of south.
- C The wind is blowing in the direction  $22.5^\circ$  east of south.
- D The wind is blowing in the direction  $18.4^\circ$  east of south.

- 30 What value of  $x$  satisfies the equation  $\log_4(x - 4) = 2$ ?

- A 5
- B 10
- C 12
- D 13

- 31 A man is standing on level ground 50 feet away from the wall of a building. He looks up at a window on the building. The angle of elevation to the bottom of the window is  $28.5^\circ$ . He then looks up at the top of the building. The angle of elevation to the top of the building is  $35^\circ$ . What is the **approximate** distance between the bottom of the window and the top of the building?

- A 5.7 feet
- B 7.9 feet
- C 8.3 feet
- D 8.5 feet



- 32 Triangle  $WXY$  has the following properties:

- The angle at vertex  $W$  is  $14^\circ$ , and the angle at vertex  $X$  is obtuse.
- The side opposite vertex  $W$  has a length of 7.00 units.
- The side opposite vertex  $X$  has a length of 9.00 units.

What is the **approximate** length of the side opposite vertex  $Y$ ?

- A 1.73 units
- B 2.08 units
- C 3.26 units
- D 5.40 units

- 33 Consider these two trigonometric functions:

$$f(x) = 3 \sin(2x) + 4$$

$$g(x) = 3 \sin\left(2x - \frac{\pi}{2}\right) + 4$$

How should the graph of  $f$  be shifted to produce the graph of  $g$ ?

- A Shift the graph of  $f$  to the left  $\frac{\pi}{4}$  units to produce the graph of  $g$ .
- B Shift the graph of  $f$  to the right  $\frac{\pi}{4}$  units to produce the graph of  $g$ .
- C Shift the graph of  $f$  to the left  $\frac{\pi}{2}$  units to produce the graph of  $g$ .
- D Shift the graph of  $f$  to the right  $\frac{\pi}{2}$  units to produce the graph of  $g$ .



34 The maximum height, in inches, a ball reaches after its first four bounces is shown in the table below.

Bounce Number	Height (in inches)
1	42.0
2	31.5
3	23.6
4	17.7

Which type of function *best* models the data and why?

- A an exponential function, because the height of the ball is decreasing by 25% with each bounce
- B an exponential function, because the height of the ball is decreasing by 75% with each bounce
- C a logistic function, because the height of the ball is decreasing by 25% with each bounce
- D a logistic function, because the height of the ball is decreasing by 75% with each bounce

35 What is the inverse function of  $g(x) = x^3 - 2$ ?

- A  $g^{-1}(x) = \sqrt[3]{x} + 2$
- B  $g^{-1}(x) = \sqrt[3]{x - 2}$
- C  $g^{-1}(x) = \sqrt[3]{x} + 2$
- D  $g^{-1}(x) = \left(\frac{x-2}{3}\right)^3$



36 What are the polar coordinates of the point  $(-2\sqrt{3}, 2\sqrt{3})$ , where  $0 \leq \theta \leq 360$ ?

- A  $(2\sqrt{6}, 150^\circ)$  and  $(-2\sqrt{6}, 210^\circ)$
- B  $(2\sqrt{6}, 135^\circ)$  and  $(-2\sqrt{6}, 315^\circ)$
- C  $(2\sqrt{6}, 120^\circ)$  and  $(-2\sqrt{6}, 240^\circ)$
- D  $(2\sqrt{6}, 30^\circ)$  and  $(-2\sqrt{6}, 330^\circ)$

37 Which equation is the rectangular form of the polar equation  $r = \frac{2}{1 + \cos\theta}$ ?

- A  $x^2 + 4y = 4$
- B  $x^2 + y^2 = 4$
- C  $y^2 + 4x = 4$
- D  $y^2 - 4x = 4$



- 38 Two parametric equations are shown below, where  $t \geq 0$ .

$$x = \frac{1}{3}\sqrt{t} + 3$$

$$y = 4t^2 - 7$$

Which nonparametric equation can be used to graph the curve described by the parametric equations?

A  $y = \frac{4}{9}(x + 1) - 7$

B  $y = \frac{4}{3}(x + 3) - 7$

C  $y = 36(x - 1)^4 - 7$

D  $y = 324(x - 3)^4 - 7$

- 39 The formula for a sequence is shown below.

$$a_n = 2a_{n-1} + 3, a_1 = 3$$

Which is another formula that represents the sequence?

A  $f(n) = 3(2^n - 1)$

B  $f(n) = 2n^3 - 3n^2 + 8n + 3$

C  $f(n) = 2(n^2 + 1)$

D  $f(n) = 3n^2 + 8n - 1$

- 40 When  $a_1 = 25,000$ , what is the sum of the infinite sequence defined by the equation  $a_{n+1} = 0.8a_n$ ?

A 125,000

B 140,000

C 160,000

D 195,000

- 41 What is the end behavior of the function  $f(x) = \frac{100}{1 + 5(0.75)^x}$ ?

A  $\lim_{x \rightarrow -\infty} f(x) = 0$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$

B  $\lim_{x \rightarrow -\infty} f(x) = 0$  and  $\lim_{x \rightarrow \infty} f(x) = 100$

C  $\lim_{x \rightarrow -\infty} f(x) = 1$  and  $\lim_{x \rightarrow \infty} f(x) = \infty$

D  $\lim_{x \rightarrow -\infty} f(x) = 1$  and  $\lim_{x \rightarrow \infty} f(x) = 100$



**AP CALCULUS — RELEASED ITEMS**

42 In the piecewise function below,  $k$  is a constant.

$$f(x) = \begin{cases} x^2 - k^2, & x \neq k \\ 4 - k, & x = k \end{cases}$$

What is the value of the limit  $\lim_{x \rightarrow k} f(x)$ ?

- A  $-2k$
- B  $2k$
- C  $0$
- D Limit does not exist.

Go to the next page.