9.3 Geometric Sequence and Series

Tuesday, May 05, 2015
10:45 AM

Geometric Sequence

A sequence in which the ratios of consecutive terms are the same. This type of sequence has a Common Ratio (r).

$$\frac{a_2}{a_1} = r \quad \frac{a_3}{a_2} = r \quad \frac{a_4}{a_3} = r \qquad r \neq 0$$

Ex. 1 State the common ratio and list the next three terms.

c) $27, 135, 675, \ldots$

$\frac{135}{27} = 5 \quad \frac{675}{135} = 5$

$r = 5$

3375, 16,875, 84,375

b) $2^n = 2, 4, 8, 16$

$r = 2$

32, 64, 128

c) $5(4)^n = 20, 80, 320, \ldots$

$\frac{320}{80} = 4 \quad \frac{80}{20} = 4$

$r = 4$

1280, 5120, 20,480

d) $n^2 = 1, 4, 9, 16, \ldots$

not geometric

The n^{th} term of a Geometric Sequence

$$a_n = a_1 r^{n-1} \qquad r \neq 0$$

a_n = geometric sequence term
a_1 = 1st term
r = common ratio

Ex. 2 Write the first five terms of the geo. seq. whose 1st term is 3 and the common ratio is 2.

$a_1 = 3 \quad r = 2$

$a_n = 3(2)^{n-1}$

$a_2 = 3(2)^{2-1} = 6$

$a_3 = 3(2)^{3-1} = 12$

$a_4 = 3(2)^{4-1} = 24$

$a_5 = 3(2)^{5-1} = 48$

3, 6, 12, 24, 48

3, 6, 12, 24, 48

Ex.3 Find the given term of the geo. sequence.

a) a_{15}: ? $a_1 = 20$, $r = 1.05$

$a_{15} = 20(1.05)^{15-1}$

$= 39.5986$

b) a_{12}: ? of 45, 15, 5, ...

$r = \frac{1}{3}$

$a_{12} = 45(1/3)^{12-1}$

$= 45(1/3)^{11}$

$= 45\left(\frac{1}{177,147}\right)$

$= \frac{45}{177,147} = \frac{5}{19,683}$

The Sum of a Finite Geometric Sequence

$$a_1 + a_1r + a_1r^2 + a_1r^3 + a_1r^4 + \ldots a_1r^{n-1}$$

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right) \qquad r \neq 1$$

Ex.4 Find the sum of the geometric sequence.

a) $\sum_{n=1}^{12} 4(0.3)^n$

$r = 0.3$

$a_1 = 4(0.3)^1$

$a_1 = 1.2$

$S_{12} = 1.2\left(\frac{1-0.3^{12}}{1-0.3}\right)$

$S_{12} = 1.7143$

b) First 12 terms of 3^n

$a_1 = 3^1$ $a_2 = 3^2$ $a_3 = 3^3$

3 9 27

$\frac{9}{3} = 3$ $\frac{27}{9} = 3$

$S_{12} = 3\left(\frac{1-3^{12}}{1-3}\right)$

$S_{12} = 797,160$

The Sum of an Infinite Geometric Sequence

IF $|r| < 1$ (less than one!) $a_1 + a_1r + a_1r^2 + a_1r^3 + \dots + a_1r^{n-1}$

$$S = \frac{a}{1-r} \qquad \sum_{k=1}^{\infty} ar^{k-1}$$

Ex. 5 Find the sum of the geometric sequence.

$3, \frac{9}{5}, \frac{27}{25}, \dots$

$\frac{\frac{27}{25}}{\frac{9}{5}} = \frac{3}{5}$ $\frac{\frac{9}{5}}{3} = \frac{3}{5}$

$r = \frac{3}{5}$

$S = \frac{a}{1-r}$

$S = \frac{3}{1-\frac{3}{5}}$

$S = 7.5$

Ex. 6 Compound Interest

A deposit of \$50 is made on the first day of each month into a savings account that pays 6% compounded monthly. What is the balance of this account at the end of 2 yrs?

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$A = 50\left(1 + \frac{.06}{12}\right)^{1 \cdot 1}$

$A = 50(1.005)$

$A_1 = 50.25$

$S_{24} = 50.25\left(\frac{1-1.005^{24}}{1-1.005}\right)$

$S_{24} = \$1277.96$

HW Tb pg 644 3-36 x3, 37, 39, 43-46, 57-83 every other odd