

9.2 Arithmetic Sequences and Series

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10:55 AM

Arithmetic Sequence

A sequence whose consecutive terms have a common difference.

$a_1, a_2, a_3, a_4, \dots, a_n$
is an arithmetic sequence if there is a number "d" (common difference) such that

$$a_2 - a_1 = d \quad a_3 - a_2 = d \quad a_4 - a_3 = d$$

8, 15, 22, ... the common difference is 7

$$22 - 15 = 7$$

$$15 - 8 = 7$$

Ex. 1 Is the sequence arithmetic? If so, why?

a) 4, 7, 10, 13, ...

$$13 - 10 = 3 \quad \text{Yes, b/c the}$$
$$10 - 7 = 3 \quad \text{common diff. is}$$
$$7 - 4 = 3$$

$$a_n = 3n + 1$$

b) 2, -3, -8, -13, ...

$$\text{Yes, b/c the}$$
$$\text{common diff is}$$

$$-5$$
$$a_n = -5n + 7$$

c) 1, 4, 9, 16, ...

No, b/c no common diff.

$$a_n = n^2$$

Ex. 2 State the next 3 terms and the common difference.

a) -5, 7, 19, ...

$$21, 43, 55$$

$$d = 12$$

b) -12, -1, 10, ...

$$21, 32, 43$$

$$d = 11$$

c) $r + 15, r + 8, r + 1, \dots$

$$d = -7$$

$$r - 6, r - 13, r - 20$$

Ex. 3

a) Show that $S_n = 3n + 5$ is an arithmetic sequence. State the first term and the common difference.

$$\begin{array}{lll} s_1 = 3(1) + 5 = 8 & 14 - 11 = 3 & s_1 = 8 \\ s_2 = 3(2) + 5 = 11 & 11 - 8 = 3 & d = 3 \\ s_3 = 3(3) + 5 = 14 & & \end{array}$$

b) Show that $a_n = 3 - 4n$ is an arithmetic sequence. State the first term and the common difference.

$$\begin{array}{lll} a_1 = 3 - 4(1) = -1 & -4 - (-5) = -4 & a_1 = -1 \\ a_2 = 3 - 4(2) = -5 & -5 - (-1) = -4 & d = -4 \\ a_3 = 3 - 4(3) = -9 & & \end{array}$$

c) Show that $a_n = \frac{1}{4}(n+3)$ is an arithmetic sequence. State the first term and the common difference.

$$\begin{array}{l} a_1 = 1 \\ d = \frac{1}{4} \end{array}$$

The n^{th} term of an Arithmetic Sequence

$$a_n = a + (n - 1)d \leftarrow \text{common difference}$$

1st term \nearrow \nwarrow term looking for

Ex. 4 Find the indicated term of the arithmetic sequence.

a) 20th term

b) 6th term

2, 6, 10, 14, ...

$$a_{20} = 2 + (20-1)(4)$$

$$a_{20} = 78$$

16, 7, -2, ...

$$a_{68} = 16 + (68-1)(-9)$$

$$a_{68} = -537$$

c) 41st term

11, 4, -3, ...

$$a_{41} = 11 + (41-1)(-7)$$

$$a_{41} = -269$$

d) Find the 12th term of

$$a_n = 8 + (n-1)6$$

$$a_{12} = 8 + (12-1)6$$

$$a_{12} = 74$$

Ex. 5

a) Find the first term of the sequence for which $a_{31} = 197$ and $d = 10$.

$$a_n = a + (n-1)d$$

$$197 = a + (31-1)(10)$$

$$197 = a + 300$$

$$a = -103$$

b) Find the first term of the sequence for which $a_{44} = 229$ and $d = -3$.

$$229 = a + (44-1)(-3)$$

$$229 = a - 344$$

$$a = 573$$

Ex 6

a) The 8th term of an arithmetic sequence is 75; the 20th term is 39. What is the common difference and a_1 ?

$$a_n = a + (n-1)d$$

$$a_8 = 75$$

$$a = 39$$

$$75 = a + (8-1)d \rightarrow 75 = a + 7d$$

$$39 = a + (20-1)d \rightarrow 39 = a + 19d$$

} system
of Eq

$$a_{20} = 39 \quad 39 = a + (20-1)d \rightarrow 39 = a + 19d \quad \text{of Eq.}$$

$$75 = a + 7d$$

$$\underline{-39 = -a - 19d}$$

$$36 = -12d$$

$$d = -3$$

$$75 = a + 7(-3)$$

$$75 = a - 21$$

$$a = 96$$

$$a_1 = 96 \text{ and} \\ d = -3$$

Hw 9.2 Tbps 635 #'s 2, 4, 10, 12, 18, 30, 36, 44,