

## 9.2 Arithmetic Sequences and Series

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10:55 AM

### Arithmetic Sequence

A sequence whose consecutive terms have a common difference.

$$a_1, a_2, a_3, a_4, \dots, a_n$$

is an arithmetic sequence if there is a number "d" (common difference) such that

$$a_2 - a_1 = d \quad a_3 - a_2 = d \quad a_4 - a_3 = d$$

8, 15, 22, ... the common difference is 7

$$22 - 15 = 7$$

$$15 - 8 = 7$$

Ex. 1 Is the sequence arithmetic? If so, why?

a) 4, 7, 10, 13, ...

13 - 10 = 3    Yes, b/c the  
10 - 7 = 3    common diff. is  
7 - 4 = 3

$$a_n = 3n + 1$$

b) 2, -3, -8, -13, ...

Yes, b/c the  
common diff is

$$a_n = -5n + 7$$

c) 1, 4, 9, 16, ...

No, b/c no common  
diff.

$$a_n = n^2$$

Ex. 2 State the next 3 terms and the common difference.

a) -5, 7, 19, ...

$$21, 43, 55$$

$$d = 12$$

b) -12, -1, 10, ...

$$21, 32, 43$$

$$d = 11$$

c)  $r+15, r+8, r+1, \dots$

$$d = -7$$

$$r-6, r-13, r-20$$

Ex. 3

a) Show that  $S_n = 3n + 5$  is an arithmetic sequence. State the first term and the common difference.

$$s_1 = 3(1) + 5 = 8$$

$$s_2 = 3(2) + 5 = 11$$

$$s_3 = 3(3) + 5 = 14$$

$$14 - 11 = 3$$

$$11 - 8 = 3$$

$$s_1 = 8$$

$$d = 3$$

b) Show that  $a_n = 3 - 4n$  is an arithmetic sequence. State the first term and the common difference.

$$a_1 = 3 - 4(1) = -1$$

$$-9 - (-5) = -4$$

$$a_1 = -1$$

$$a_2 = 3 - 4(2) = -5$$

$$-5 - (-1) = -4$$

$$d = -4$$

$$a_3 = 3 - 4(3) = -9$$

c) Show that  $a_n = \frac{1}{4}(n+3)$  is an arithmetic sequence. State the first term and the common difference.

$$a_1 = 1$$

$$d = \frac{1}{4}$$

The  $n^{\text{th}}$  term of an Arithmetic Sequence

$$a_n = a + (n-1)d \leftarrow \text{common difference}$$

1<sup>st</sup> term  $\nearrow$

$\nwarrow$  term looking for

Ex. 4 Find the indicated term of the arithmetic sequence.

a) 20<sup>th</sup> term

$$2, 6, 10, 14, \dots$$

$$a_{20} = 2 + (20-1)(4)$$

$$a_{20} = 78$$

b) 68<sup>th</sup> term

$$16, 7, -2, \dots$$

$$a_{68} = 16 + (68-1)(-9)$$

$$a_{68} = -587$$

c) 41<sup>st</sup> term

$$11, 4, -3, \dots$$

$$a_{41} = 11 + (41-1)(-7)$$

$$a_{41} = -269$$

d) Find the 12<sup>th</sup> term of

$$a_n = 8 + (n-1)6$$

$$a_{12} = 8 + (12-1)6$$

$$a_{12} = 74$$

### Ex. 5

- a) Find the first term of the sequence for which  $a_{31} = 197$  and  $d = 10$ .

$$\begin{aligned} a_n &= a + (n-1)d \\ 197 &= a + (31-1)(10) \\ 197 &= a + 300 \\ a &= -103 \end{aligned}$$

- b) Find the first term of the sequence for which  $a_{44} = 229$  and  $d = -8$ .

$$\begin{aligned} 229 &= a + (44-1)(-8) \\ 229 &= a_1 - 344 \\ a_1 &= 573 \end{aligned}$$

### Ex 6

- a) The 8<sup>th</sup> term of an arithmetic sequence is 75; the 20<sup>th</sup> term is 39. What is the common difference and  $a_1$ ?

$$\begin{aligned} a_n &= a + (n-1)d \\ a_8 &= 75 & 75 &= a + (8-1)d \rightarrow 75 = a + 7d \\ a_{20} &= 39 & 39 &= a + (20-1)d \rightarrow 39 = a + 19d \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{system of Eq.}$$

$$\begin{array}{r} 75 = a + 7d \\ -39 = -a - 19d \\ \hline 36 = -12d \\ d = -3 \end{array} \quad \begin{array}{l} \curvearrowright \\ 75 = a + 7(-3) \\ 75 = a - 21 \\ a = 96 \end{array}$$

$$\begin{aligned} a_1 &= 96 \text{ and} \\ d &= -3 \end{aligned}$$

HW 9.2 Tb ps 635 #'s 2, 4, 10, 12, 18, 30, 36, 44,

## The Sum of a Finite Arithmetic Sequence

$$S_n = \frac{n}{2} (a_1 + a_n)$$

only works for arithmetic sequences

$a_n$ : arithmetic sequence (last term)  
 $a_1$ : the first term

### Ex. 7

a) Find the sum of the 1<sup>st</sup> 63 terms of the series

$$a_n = -19, -13, -7, \dots$$

$\begin{array}{cc} \vee & \vee \\ +6 & +6 \end{array}$

$$a_n = a + (n-1)d$$

$$a_{63} = -19 + (63-1)(6)$$

$$a_{63} = 353$$

$$S_{63} = \frac{63}{2}(-19 + 353)$$

$$S_{63} = 10,521$$

b) Find the sum of the integers from 1 to 100.

$$1+2+3+\dots+100$$

$$a_1 = 1 \quad d = 1$$

$$a_{100} = 100$$

$$S_{100} = \frac{100}{2}(1+100)$$

$$S_{100} = 5050$$

### Ex. 8

An auditorium has 30 rows of seats. There are 20 seats in the first row, 21 seats in row 2, 22 seats in row 3, and so on. How many seats are there in all 30 rows?

$$a_n = a + (n-1)d$$

$$a_{30} = 20 + (30-1)(1)$$

$$a_{30} = 49$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{30} = \frac{30}{2}(20 + 49)$$

$$S_{30} = 1035 \text{ seats}$$

Hw Tb pg. 635 60, 64, 66, 72, 79, 82, 84, 86