9.1 Sequences and Series

Wednesday, April 29, 2015
10:23 AM

Sequence

Is a collection that is ORDERED.

- ordered from first to last
- is a function whose domain is the Set of Positive Integers.
- written with subscript notation $(a_n = 2n+1)$

Infinite Sequence

A function whose domain is the set of positive integers. The function values: $a_1, a_2, a_3, a_4, \ldots, a_n$ are the terms of the sequence. If the domain of the function consists of the first n positive integers ONLY, the sequence is called Finite Sequence.

Ex. 1 Find the first six terms of each sequence.

a) $a_n = 3n-2$

$a_1 = 3(1)-2 = 1$ $\;\rangle +3$

$a_2 = 3(2)-2 = 4$ $\;\rangle +3$

$a_3 = 3(3)-2 = 7$ $\;\rangle +3$

$a_4 = 3(4)-2 = 10$ $\;\rangle +3$

$a_5 = 3(5)-2 = 13$ $\;\rangle +3$

$a_6 = 3(6)-2 = 16$

b) $a_n = \frac{n-1}{n}$

$a_1 = \frac{1-1}{1} = 0$

$a_2 = \frac{2-1}{2} = 1/2$

$a_3 = \frac{3-1}{3} = 2/3$

$a_4 = 4-1/4 = 3/4$

$a_5 = 5-1/5 = 4/5$

$a_6 = 6-1/6 = 5/6$

c) $b_n = (-1)^{n-1}\left(\frac{2}{n}\right)$

d) $c_n = \begin{cases} n & \text{if } n \text{ is even} \\ \frac{1}{n} & \text{if } n \text{ is odd} \end{cases}$

$b_1 = 2$
$b_2 = -1$
$b_3 = 2/3$
$b_4 = -1/2$
$b_5 = 2/5$
$b_6 = -1/3$

$c_1 = 1$
$c_2 = 2$
$c_3 = 1/3$
$c_4 = 4$
$c_5 = 1/5$
c_6 6

In each example above, you were given the "n^{th} term", which is the formula. The listing of a few terms (like above) is not SUFFICIENT to define a sequence, the "n term" must be written.

Ex. 2 Determine the n^{th} term of each sequence.
* YOU HAVE TO THINK! LOOK FOR PATTERNS!

a) $1, 3, 5, 7, \ldots$

$a_1 = 1$ $\nearrow +2$
$a_2 = 3$ $\nearrow +2$
$a_3 = 5$ $\nearrow +2$
$a_4 = 7$
$a_n = 2n - 1$

n =	1	2	3	4
terms:	1	3	5	7

b) $2, 5, 10, 17, \ldots$

$a_1 = 2$ $\nearrow +3$
$a_2 = 5$ $\nearrow +5$
$a_3 = 10$
$a_4 = 17$
$a_n = n^2 + 1$

c) $1, 1/3, 1/9, 1/27$

$$a_n = \frac{1}{3^{n-1}}$$

d) $e, \frac{e^2}{2}, \frac{e^3}{3}, \frac{e^4}{4}$

$$a_n = \frac{e^n}{n}$$

When signs of a sequence alternate, you use factors such as: $(-1)^{n+1}$ which equals 1 if n is odd and -1 if n is even

$(-1)^n$ which equals -1 if n is odd and 1 if n is even

$1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}$

$a_n = (-1)^{n+1}(\frac{1}{n})$

$-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{5}$

$a_n = (-1)^n(\frac{1}{n})$

Recursive Sequence

- need to be given one or more of the first few terms.
- all other terms are defined using previous terms.

ie Fibonacci Sequence

$1, 1, 2, 3, 5, 8, 13, \ldots \longrightarrow a_k = a_{k-2} + a_{k-1}$

Terms 1 2 3 4 5 6 7

$a_3 = a_{k-2} + a_{k-1}$
$= a_{3-2} + a_{3-1}$
$= a_1 + a_2$
$= 1 + 1$
$a_3 = 2$

$a_8 = a_{8-2} + a_{8-1}$
$= a_6 + a_7$
$= 8 + 13$
$a_8 = 21$

Ex. 3 Write the first 5 terms of the following recursive sequences.

a) $s_1 = 1$; $s_n = 4s_{n-1}$

$s_2 = 4s_{2-1} \rightarrow 4s_1 \rightarrow 4(1) = 4$
$s_3 = 4s_{3-1} \rightarrow 4s_2 \rightarrow 4(4) = 16$
$s_4 = 4s_{4-1} \rightarrow 4s_3 \rightarrow 4(16) = 64$
$s_5 = 4s_{5-1} \rightarrow 4s_4 \rightarrow 4(64) = 256$

b) $f_1 = 1$; $f_n = nf_{n-1}$

$f_2 = 2f_{2-1} \rightarrow 2f_1 \rightarrow 2(1) = 2$
$f_3 = 3f_{3-1} \rightarrow 3f_2 \rightarrow 3(2) = 6$
$f_4 = 4f_{4-1} \rightarrow 4f_3 \rightarrow 4(6) = 24$
$f_5 = 5f_{5-1} \rightarrow 5f_4 \rightarrow 5(24) = 120$

Factorial (#!)

A special type of Product

Special case

$0! = 1$

$1! = 1$

$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots (n-1) \cdot n$

$n! = n(n-1)(n-2)(n-3)\ldots \quad n \geq 2$

$0! = 1$

$1! = 1$

$2! = 1(2) = 2$

$3! = 1(2)(3) = 6$

$4! = 1(2)(3)(4) = 24$

$5! = 1(2)(3)(4)(5) = 120$

$3 \cdot 5!$

$3 \cdot 5! = 3(1 \cdot 2 \cdot 3 \cdot 4 \cdot 5)$

$= 3(120)$

$= 360$

Factorials follow the same conventions for order of operations as do exponents.

Ex. 4 Write the first 5 terms; begin with $n=0$!

a) $a_n = \frac{2^n}{n!}$

$a_0 = \frac{2^0}{0!} = \frac{1}{1} = 1$

$a_1 = \frac{2^1}{1!} = \frac{2}{1} = 2$

$a_2 = \frac{2^2}{2!} = \frac{4}{2} = 2$

$a_3 = \frac{2^3}{3!} = \frac{8}{6} = \frac{4}{3}$

$a_4 = \frac{2^4}{4!} = \frac{16}{24} = \frac{2}{3}$