6.4 Vectors and Dot Products

Tuesday, April 21, 2015
10:03 AM

Dot Product

a type of vector operation; the product yields a scalar rather than a vector.

Defn. of Dot Product

The dot product of $u = \langle u_1, u_2 \rangle$ and $v = \langle v_1, v_2 \rangle$ is

$$u \cdot v = u_1 v_1 + u_2 v_2$$

ie $u = \langle 2, 4 \rangle$ $v = \langle -3, 5 \rangle$

$$u \cdot v = 2(-3) + 4(5)$$
$$= 14$$

Properties of the Dot Product

Let u, v, and w be vectors in the plane or in space and let c be a scaler.

1) $u \cdot v = v \cdot u$
2) $0 \cdot v = 0$
3) $u \cdot (v + w) = uv + uw$
4) $v \cdot v = ||v||^2$
5) $c(u \cdot v) = cu \cdot v$ or $u \cdot cv$

Ex.1 Find the Dot Product

a) $\langle 4, 5 \rangle \cdot \langle 2, 3 \rangle$

$4(2) + 5(3)$

23

b) $\langle 2, -1 \rangle \cdot \langle 1, 2 \rangle$

$2(1) + -1(2)$

0

c) $\langle 0, 3 \rangle \cdot \langle 4, -2 \rangle$

$0(4) + 3(-2)$

-6

Scalars

Ex.2 Using Properties of Dot Products

$u = \langle -1, 3 \rangle$ $v = \langle 2, -4 \rangle$ $w = \langle 1, -2 \rangle$

a) $(u \cdot v)w$

$(-1(2) + 3(-4)) \langle 1, -2 \rangle$

$-14 \langle 1, -2 \rangle$

b) $u \cdot 2v$

$\langle -1, 3 \rangle \cdot 2 \langle 2, -4 \rangle$

$\langle -1, 3 \rangle \cdot \langle 4, -8 \rangle$

$\langle -14, 28 \rangle$
Vector

$(-1)(4) + 3(-8)$
-28
Scalar

Ex.3 Dot Product and Length

The dot product of U with itself is 5. What is the length of U?

$U \cdot U = 5$

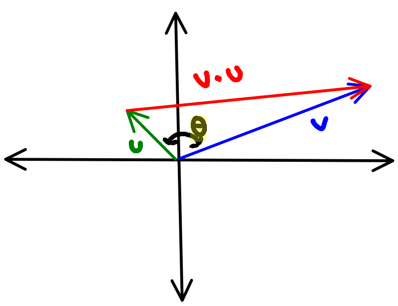
$U \cdot U = \|U\|^2$

$\sqrt{5} \cdot \sqrt{5} = 5$

$\|5\|^2 =$

length is $\sqrt{5}$

Angle between two nonzero vectors is the angle θ, $0 \le \theta \le \pi$ between its respective STANDARD POSITION VECTORS.



* Angle between zero vector and another vector is undefined.

Angle between two vectors

$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|}$$

Ex.4 Find the angle.

a) $u = \langle 4,3 \rangle$ $v = \langle 3,5 \rangle$

$$\cos\theta = \frac{\langle 4,3 \rangle \cdot \langle 3,5 \rangle}{\|\langle 4,3 \rangle\| \, \|\langle 3,5 \rangle\|}$$

$\cos\theta = 4(3) + 3(5)$

$$\left(\sqrt{(4)^2+(3)^2}\right)\left(\sqrt{(3)^2+(5)^2}\right)$$

$$\cos\theta = \frac{27}{5\sqrt{34}}$$

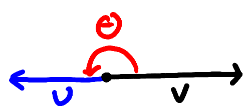
$$\theta = \arccos \frac{27}{5\sqrt{34}} \qquad \cos^{-1}(27/(5\sqrt{34}))$$

$$\theta \approx 22.2^\circ$$

Alternate Form

$$u \cdot v = \|u\|\|v\|\cos\theta$$

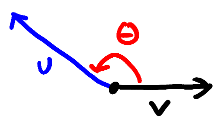
5 possible orientations of 2 vectors



$\theta = \pi$

$\cos\theta = -1$

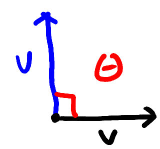
(opposite direction)



$\frac{\pi}{2} < \theta < \pi$

$-1 < \cos\theta < 0$

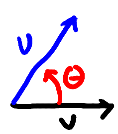
obtuse ∠



$\theta = \frac{\pi}{2}$

$\cos\theta = 0$

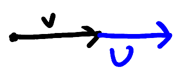
90° ∠



$0 < \theta < \pi/2$

$0 < \cos\theta < 1$

Acute ∠



$\theta = 0$

$\cos\theta = 1$

Same direction

Defn of Orthogonal Vectors

Vectors u and v are orthogonal vectors if

$$u \cdot v = 0$$

Orthogonal is basically perpendicular (forms a 90° angle)

* A zero vector is orthogonal to every vector u, because $0 \cdot u = 0$.

To be parallel, the vector components have to be equal

or $u = kv$

$u = \langle 2, 4 \rangle$

$v = \langle 6, 12 \rangle \rightarrow 3\langle 2, 4 \rangle$

$u = kv$

Ex. 5 Determine if the vectors are Orthogonal vectors, parallel or neither.

a) $u = \langle 2, -3 \rangle$ $v = \langle 6, 4 \rangle$

$\langle 2, -3 \rangle \cdot \langle 6, 4 \rangle$

$2(6) + -3(4)$

0

Orthogonal vectors

b) $u = 8i + 4j$

$v = -2i - j$

$\langle 8, 4 \rangle \cdot \langle -2, -1 \rangle$

$8(-2) + 4(-1)$

-20 Not orthogonal

$u = v$?

$\langle 8, 4 \rangle = \langle -2, -1 \rangle$

$-4 \langle -2, -1 \rangle = \langle -2, -1 \rangle$

Parallel vectors