6.3 (continued)

Monday, April 20, 2015
9:39 AM

The unit vectors $\langle 1,0\rangle$ and $\langle 0,1\rangle$ are called Standard Unit Vectors and are denoted by:

$$\mathbf{i}=\langle 1,0\rangle \quad \text{and} \quad \mathbf{j}=\langle 0,1\rangle$$

* **i** is written **Bold Face** to distinguish from i the imaginary unit. *

Can be used to represent any vector $v=\langle v_1, v_2\rangle$

$$\begin{aligned} v &= \langle v_1, v_2\rangle \\ &= v_1\langle 1,0\rangle + v_2\langle 0,1\rangle \\ &\quad \underbrace{v_1 \mathbf{i} + v_2 \mathbf{j}} \end{aligned}$$

Scalars are horizontal and vertical components of v (respectively).

The vector sum $v_1\mathbf{i} + v_2\mathbf{j}$ is called a Linear Combination or a Position Vector.

Suppose $\vec{v}$ is a vector with initial point $P_1(x_1, y_1)$, not at the origin, and the terminal point $P_2(x_2, y_2)$.
If $\vec{v} = \overrightarrow{P_1P_2}$, then $\vec{v}$ is equal to the position vector:

$$\vec{v} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j}$$

Standard Form

$v = a\mathbf{i} + b\mathbf{j}$; with initial point $(0,0)$ and terminal point (a,b)

$$\begin{aligned} v &= (a-0)\mathbf{i} + (b-0)\mathbf{j} \\ &= a\mathbf{i} + b\mathbf{j} \end{aligned}$$

Ex. 5

a) Let $\vec{v}$ be the vector with initial point $(-4,3)$ and terminal point $(1,4)$. Write $\vec{v}$ as a position vector of unit vectors $\mathbf{i}$ and $\mathbf{j}$.

$\vec{U} = (1-(-4))i + (4-3)j$
$= 5i + 1j$ Position Vector

SAME $5\langle 1,0\rangle + 1\langle 0,1\rangle = \langle 5,0\rangle + \langle 0,1\rangle = \langle 5,1\rangle$

b) $P_1 = (-3,6)$ $P_2 = (5,3)$

$\vec{U} = (5-(-3))i + (3-6)j$
$= 8i - 3j$

Equality of Vectors

Two vectors v and w are EQUAL if and only if their corresponding components are equal.

IF $v = a_1 i + b_1 j$ and $w = a_2 i + b_2 j$ then
$\vec{v} = \vec{w}$ if $a_1 = a_2$ $b_1 = b_2$

Ex. 6 Add or subtract $\vec{v} = 2i + 3j$ $\vec{w} = 3i - 4j$

a) $\vec{v} + \vec{w}$
$= (2i + 3j) + (3i - 4j)$
$= 5i - 1j$

b) $\vec{v} - \vec{w}$
$= (2i + 3j) - (3i - 4j)$
$= (2i + 3j) + (-3i + 4j)$
$= -1i + 7j$

c) $2\vec{v} - 3\vec{w}$
$2(2i + 3j) - 3(3i - 4j)$
$(4i + 6j) - (9i - 12j)$
$(4i + 6j) + (-9i + 12j)$
$\boxed{-5i + 18j}$

d) Unit Vector for $\vec{v} = 4i - 3j$

magnitude
$\|v\| = \sqrt{(4)^2 + (-3)^2}$
$= \sqrt{16+9}$
$= \sqrt{25}$
$= 5$

$\vec{U} = \frac{\vec{v}}{\|v\|}$

$\vec{U} = \frac{4i - 3j}{5}$

$\boxed{\frac{4i}{5} - \frac{3j}{5}}$

e) Unit vector for $\vec{v} = -3j$

$\|v\| = \sqrt{(0)^2 + (-3)^2}$

$\vec{U} = -3j$

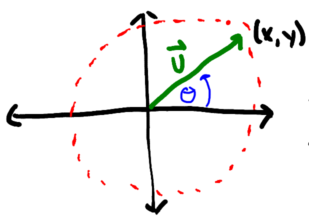
$= \sqrt{9}$ $\qquad\qquad\qquad\qquad \frac{\quad}{3}$

$= 3$ $\qquad\qquad\qquad\qquad = -j$

Direction Angles

θ is the direction angle of $\vec{u}$

$\vec{u}$ is a unit vector such that θ is the angle, measured counter clockwise, from the positive x-axis to $\vec{u}$; the terminal point of $\vec{u}$ lies on the unit circle.



$y = \sin\theta$

$x = \cos\theta$

Component

$u = \langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle$

position

$(\cos\theta)i + (\sin\theta)j$

θ is the direction angle of vector u.

If u is the vector with direction angle θ and if v is any vector that makes an angle θ with the positive x-axis, then it has the same direction as u and

$$\vec{v} = \|v\| \langle \cos\theta, \sin\theta \rangle$$

$$= \|v\|(\cos\theta)i + \|v\|(\sin\theta)j$$

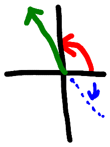
* θ is found using $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\left(\frac{y}{x}\right)$ Direction Angle

Ex.7 Determine the direction angle and magnitude

a) $\vec{v} = 3\cos 45^\circ + 3\sin 45^\circ$

$\|v\| = 3 \qquad \theta = 45^\circ$

b) $\vec{v} = -2i + 5j$



Direction

$\tan\theta = \frac{5}{-2}$

$\theta = -68.2^\circ$

$\theta = 180 + (-68.2^\circ)$

$= 111.8^\circ$

Magnitude

$||v|| = \sqrt{(-2)^2 + (5)^2}$

$= \sqrt{29}$

$\vec{v} = \sqrt{29}\cos(111.8^\circ)i + \sqrt{29}\sin(111.8^\circ)j$

$= \langle \sqrt{29}\cos(111.8), \sqrt{29}\sin(111.8) \rangle$

Ex 8 Find the component of v given its magnitude and direction angle

a) $||v|| = 4$

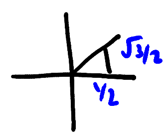
$\theta = 60$

$4\cos 60 i + 4\sin 60 j$

$\langle 4\cos 60, 4\sin 60 \rangle$

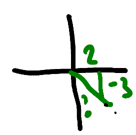
$\langle 4(1/2), 4(\sqrt{3}/2) \rangle$

$\langle 2, 2\sqrt{3} \rangle$



b) $||\vec{v}|| = 5$; $\vec{v}$ is in the direction of $2i - 3j$

$\tan \frac{-3}{2}$



$\vec{v} = 5\cos(-56.3)i + 5\sin(-56.3)j$

$= \langle 5\cos(-56.3), 5\sin(-56.3) \rangle$

$\theta = -56.3^\circ$

$= \langle 2.77, -4.16 \rangle$

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