6.3 Vectors in the Plane

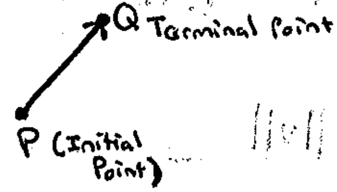
Many quantities (i.e. distance, time, temperature...) can be represented by a REAL NUMBER. However, there are quantities (i.e. force, velocity, acceleration...) that cannot be represented by a single number.

These types of quantities use Vectors to represent them.

Vector

Is a directed line segment that has two and only two defining characteristics: 1) magnitude (size / quantity) 2) direction

$\overrightarrow{PQ}$



- The magnitude of a vector is the distance from P to Q; if coordinates are given use the distance formula to find it. $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$
- A vector's magnitude is denoted by: $\|PQ\|$ or $|\overrightarrow{PQ}|$

  Not to be confused with Abs. Value $|x|$ or greatest integer $[\![x]\!]$
- Vectors can be represented by **bold face** lower case letters such as **u, v, w.**
- Two vectors are EQUIVALENT if they have the same magnitude and direction, $V = W$
- Use the Slope to find/compare the direction of vectors. $m = \frac{y_2-y_1}{x_2-x_1}$
- Zero Vector, "0," a vector that has a magnitude of 0 and no direction. This vector has both the initial and terminal points at the origin. It is denoted by $0 = \langle 0, 0 \rangle$.

Ex. 1

Let **u** be represented by the vector from P(0, 0) to Q(4, 6) and let **v** be represented by the vector from R(2,3) to S(6, 9). Show that the vectors are equivalent.

Magnitude

$\|PQ\| = \sqrt{(4-0)^2 + (6-0)^2} = \sqrt{52} = 2\sqrt{13}$ or 7.21

$\|RS\| = \sqrt{(6-2)^2 + (9-3)^2} = \sqrt{52}$

Both vectors have the same magnitude

Direction

$m\,\overrightarrow{PQ} = \frac{6-0}{4-0} = \frac{6}{4} = \frac{3}{2}$

$m\,\overrightarrow{RS} = \frac{9-3}{6-2} = \frac{6}{4} = \frac{3}{2}$

Both vectors have the same direction

$U = V$

A vector is in Standard Position when its initial point is at the origin. A vector in STANDARD POSITION can be uniquely represented by the coordinates of it terminal point (v_1, v_2). This is the component form of vector **v**; written as $V = \langle v_1, v_2 \rangle$. (The coordinates v_1 and v_2 are the components of **v**.)

6.3 Vectors in the Plane

Component Form of a Vector

The component form of the vector with initial point $P(p_1, p_2)$ and terminal point $Q(q_1, q_2)$ is:

$$\overrightarrow{PQ} = \langle q_1 - p_1 \, , \, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}$$

The magnitude (length) of v is: $\|v\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} \rightarrow \sqrt{v_1^2 + v_2^2}$

If $\|v\|$ = 1, $\vec{v}$ is a unit vector. $\|v\|$ = 0, if and only if v is the zero vector 0.

Two vectors $\vec{U}$ = $\langle u_1, u_2 \rangle$ and $\vec{V}$ = $\langle v_1, v_2 \rangle$ are **EQUAL** if and only if $u_1 = v_1$ and $u_2 = v_2$.

Ex. 2

Find the component from, length, and direction of the vector v that that has the initial point (3, −6) and a terminal point (-4, 5).

Component Form

$V = \langle -4-3 \, , \, 5+6 \rangle$

$V = \langle -7, 11 \rangle$

Magnitude

$\|v\| = \sqrt{(-7)^2 + (11)^2}$

$\|v\| = \sqrt{170}$ or 13.04

Direction

$\dfrac{\text{rise}}{\text{run}}$

$\dfrac{11}{-7}$

Vector Operations

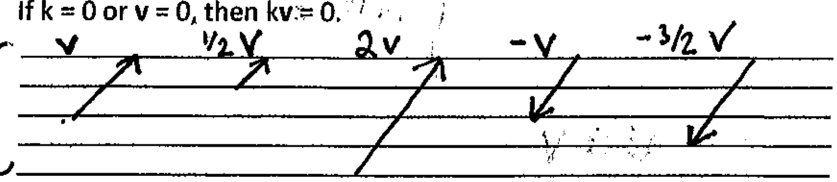
Two basic operations: 1) Scalar multiplication

2) vector addition

Scalar multiplication: the product of a vector v and a scalar k is the vector that is $|k|$ as long as v.

- If $k > 0$, kv has the same direction as v, and a magnitude of kv.
- If $k < 0$, kv has the opposite direction as v, and a magnitude of kv.
- If k = 0 or v = 0, then kv = 0.

Scalar mult. Geometric



Component Form definition of Scalar Multiplication

$U = \langle U_1, U_2 \rangle$

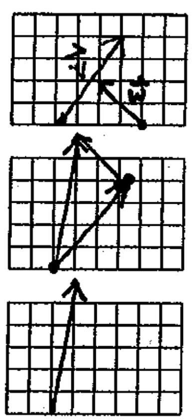
$k\vec{U} = k\langle U_1, U_2 \rangle \longrightarrow \langle kU_1, kU_2 \rangle$

6.3 Vectors in the Plane

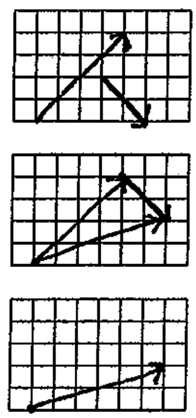
Vector Addition

1. Position the vectors so that the initial point of one coincides with the terminal point of the other.
2. The sum of **u** + **v** is formed by joining the initial point of the second vector **v** with the terminal point of the first vector **u**.

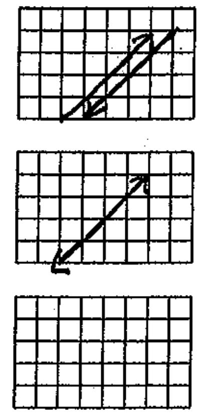
a) $\vec{v}+\vec{w}$



b) $\vec{w}+\vec{v}$



c) $\vec{v}+(-\vec{v})=0$



Component Form definition of Vector Addition

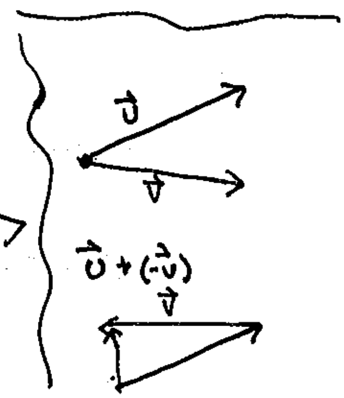
$$U=\langle u_1, u_2\rangle \quad V=\langle v_1, v_2\rangle$$

$$\vec{u}+\vec{v}=\langle u_1+v_1,\ u_2+v_2\rangle$$

Negative of $\vec{v}=\langle v_1, v_2\rangle$ is $-V=(-1)V=\langle -v_1, -v_2\rangle$

Difference of U and V is

$$U-V=U+(-V)=\langle u_1-v_1, u_2-v_2\rangle$$



Ex. 3

Let v = $\langle -2, 5\rangle$ and w = $\langle 3, 4\rangle$, find each of the following vectors:

a) $2v$

$=2\langle -2, 5\rangle$

$=\langle -2(2), 5(2)\rangle$

$=\langle -4, 10\rangle$

b) $w-v$

$=\langle 3-(-2), 4-5\rangle$

$=\langle 5, -1\rangle$

c) $v+2w$

$=\langle -2+3(2), 5+4(2)\rangle$

$=\langle -2+6, 5+8\rangle$

$=\langle 4, 13\rangle$

See TB pg 447 for a geometric sketch of v+2w