6.1 Law of Sines

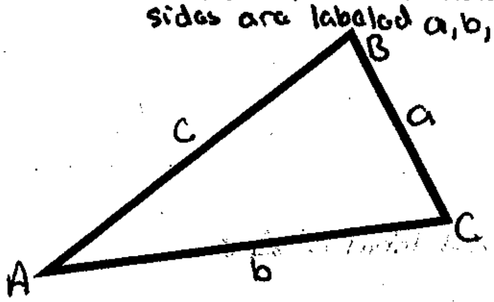
e laws of **SINES** and **COSINES** are used to solve triangles that **ARE NOT RIGHT TRIANGLES.**

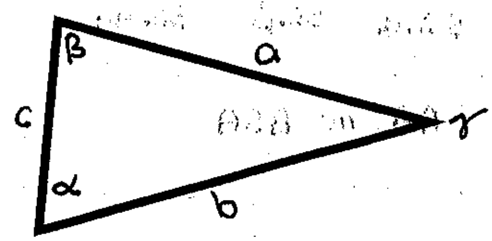
Oblique Triangle : is a triangle that has **NO RIGHT ANGLE**. The following are two types of OBLIQUE TRIANGLES:

Acute Triangle : is a triangle that has all **ACUTE ANGLES** (0 < *m* < 90).

Obtuse Triangle : is a triangle that has one **OBTUSE ANGLE** (*m* > 180).

* As a standard notation, the angles of a Δ are labeled A, B, C, and the opposite sides are labeled a, b, c.

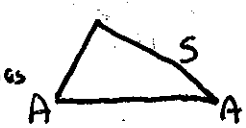


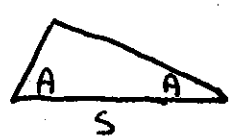


To solve an **oblique triangle** you need to know the measure of **at least** one side and any two other parts of the triangle. en solving, determine the measure of any angle or side that is not given to you. There are four cases when solving oblique triangles:

Law of Sines

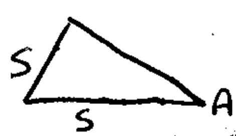
Case 1: One side and two angles are known. SAA or ASA





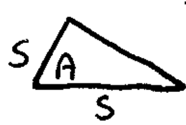
Case 2: Two sides and an angle **opposite** one of the sides are known. SSA

No there is not an ASS case!

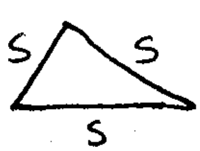


Law of Cosines

Case 3: Two sides and the included angle are known. SAS



Case 4: Three sides are known. SSS

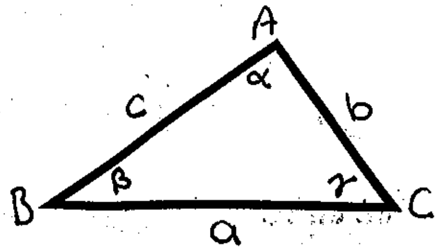


or **LAW OF SINES** you will use CASE 1 and CASE 2 and for **LAW OF CONSINES** you will use Case 3 and Case 4.

6.1 Law of Sines

Law of Sines

For a triangle with sides ***a***, ***b***, and ***c*** and opposite angles respectively,



Proportions

Use when finding angle measures:

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

FYI

Calculator must be in deg mode!

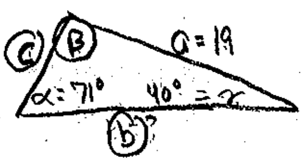
Use when finding length of sides:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Ex. 1: CASE 1 SAA or ASA

a) Solve the triangle when: $\alpha = 71°$, $\gamma = 40°$, and a = 19 cm. * Draw and label a Δ *

SAA



$\measuredangle B$

$180-(71+40)$

$\boxed{69°}$

Side b

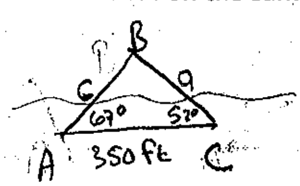
$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$ $\frac{19}{\sin 71} = \frac{b}{\sin 69}$

$\boxed{b = 18.76\text{ cm}}$

$\frac{19}{\sin 71} = \frac{c}{\sin 40}$

$\boxed{c = 12.92\text{ cm}}$

b) Two points ***A*** and ***B*** are on opposite sides of a river. Point ***C*** is located 350 ft from ***A*** on the same side of the river as ***A***. In Δ ABC, m< C = 52° and m< A = 67°.



1) Determine the measure of angle **B**.

$m\measuredangle B = 180-(67+52)$

$= \boxed{61°}$

ASA

2) Determine the distance between **A** and **B**, **B** and **C**.

A and B

Side c

$\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{350}{\sin 61} = \frac{c}{\sin 52}$

$c \approx 315.34$ ft

B and C

Side a

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$\frac{350}{\sin 61} = \frac{a}{\sin 67}$

$a \approx 368.36$ ft

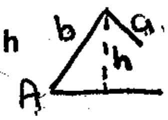
6.1 Law of Sines

Ambiguous Case "SSA" (aka Case 2)

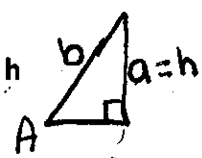
The "given" or "known" information may result in 1 triangle, 2 triangles, or NO triangles at all. Suppose that we are given sides "*a*" and "*b*", and measure < **A**; the key to determine the possible triangles, IF ANY, that may be formed lies primarily with the **height, *h***, and the fact that: ***h* = *b*sinA** Find h

Angle **A** is acute:

No Δ: if a < h



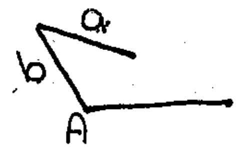
One Δ: if a = h



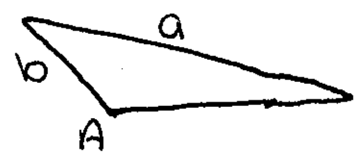
Two Δ's: if h < a < b

Angle **A** is obtuse:

No Δ: if a ≤ b



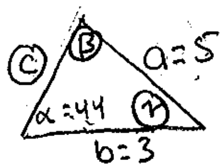
One Δ: if a > b



Ex. 2:

a) Solve the triangle when α = 44°, a = 5, and b = 3.

h = bsinA
h = 3sin44
h ≈ 2.08



∡B

$\frac{\sin 44}{5} = \frac{\sin B}{3}$

$\sin B = \frac{3\sin 44}{5}$

$\sin B = .4168$

$\sin^{-1}(.4168)$

$\boxed{B_1 = 24.6°}$

∡C

$180 - (44 + 24.63)$

$\boxed{111.4°}$

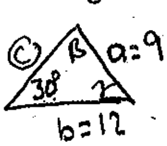
Side C

$\frac{5}{\sin 44} = \frac{c}{\sin 111.4}$

$\boxed{C \approx 6.70}$

b) Solve the triangle when α = 30°, a = 9, and b = 12.

h = bsinA
h = 12 sin 30
h = 6
h < a < b
6 < 9 < 12
‖ 2 Δ's ‖



∡B

$\frac{\sin 30}{9} = \frac{\sin B}{12}$

$\sin B = .6667$

$\boxed{B_1 = 41.8°}$

$\gamma_1 = 180 - (30 + 41.8) = \boxed{108.2°}$

β_2

$180 - 41.8°$

$\boxed{B_2 = 138.2°}$

γ_2

$180 - (30 + 138.2)$

$\boxed{11.8°}$

Side C

C_1

$\frac{9}{\sin 30} = \frac{C_1}{\sin 108.2}$

$\boxed{C_1 = 17.10}$

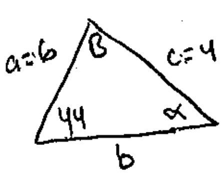
C_2

$\frac{9}{\sin 30} = \frac{C_2}{\sin 11.8}$

$\boxed{C_2 = 3.68}$

c) Solve the triangle when γ = 44°, a = 6, and c = 4.

h =



h = 6sin44
h = 4.17
c < h

No Solution

$\frac{\sin 44}{4} = \frac{\sin \alpha}{6}$

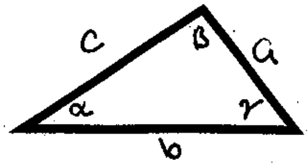
$\sin \alpha = 1.1491$

Domain error
-1 to 1

6.1 Law of Sines

Area of an Oblique Triangle

(see Tb pg 432 for explanation)

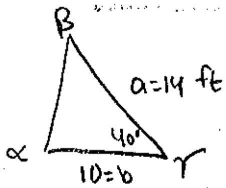


The area "A" of ΔABC is:

$A = \frac{1}{2}ab\sin\gamma$ or $A = \frac{1}{2}bc\sin\alpha$ or $A = \frac{1}{2}ac\sin\beta$

Ex. 3:

Find the area of the triangle for which a = 14 ft, b = 10 ft, and γ = 40°.



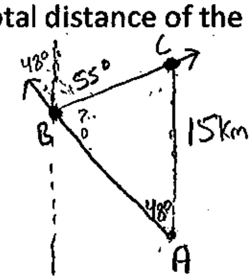
$A = \frac{1}{2}(14)(10)\sin 40$

$A = 44.995 \text{ ft}^2$

$\boxed{A = 45 \text{ ft}^2}$

Ex. 4:

The course for a boat race starts at point A and proceeds in the direction of N 48° W to point B, then in the direction N 55° E to point C, and finally back to A. Point C lies 15 km directly north of point A. Approximate the total distance of the race course. DRAW A PICTURE!



$\measuredangle B$: $180-(55+48)$, $\measuredangle B = 77^\circ$

$\measuredangle C$: $180-(77+48)$, $\measuredangle C = 55^\circ$

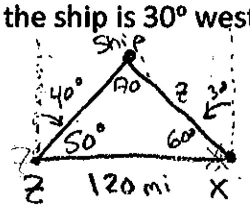
Side c: $\frac{c}{\sin 55} = \frac{15}{\sin 77}$, $c \approx 12.61$ Km

Side a: $\frac{a}{\sin 48} = \frac{15}{\sin 77}$, $a \approx 11.44$ km

$\boxed{\text{distance is } 39.05 \text{ km}}$

Ex. 5:

Coast Guard Station Zulu is located at 120 mi due west of Station X-Ray. A ship at sea sends an SOS (distress call) that is received by each station. The call to Station Zulu indicates that the location of the ship is 40° east of north. The call to station X-Ray indicates that the location of the ship is 30° west of north.



1) How far is each station from the ship?

X-ray distance: $\frac{120}{\sin 70} = \frac{z}{\sin 50}$, 97.82 mi

Zulu distance: $\frac{120}{\sin 70} = \frac{x}{\sin 60}$, 110.59 mi

2) If a helicopter capable of flying 200 mi/hr is dispatched from the nearest station to the ship, how long will it take the helicopter to reach the ship?

$d = rt$

$97.82 = 200t$

$t = .49$ hrs → $.49 \cdot 60$ → 29.4 minutes