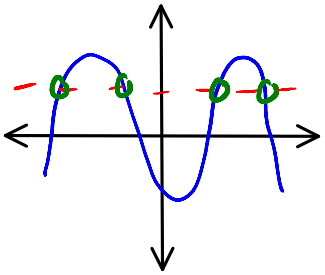
4.7 Inverse Trig Functions

Wednesday, April 01, 2015
11:01 AM

For a function to have an **Inverse** that is a function, the **Original** function must pass the HORIZONTAL LINE TEST!

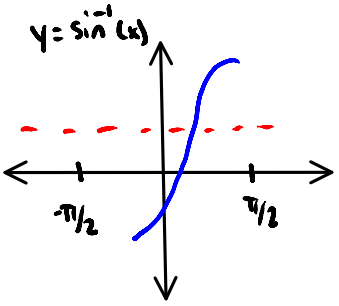
ALL TRIG FUNCTIONS FAIL THE HLT!

$y = \sin x$



Fails HLT

$y = \sin^{-1}(x)$



Passes HLT

With trig functions, you must restrict the domain so that you have **inverse trig functions**.

Defn. of Inverse Sine Function

$y = \arcsin x$ or $y = \sin^{-1} x$

The inverse of sine is defined if and only if $\sin y = x$; where $-1 \le x \le 1$

$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

x and y switch places for inverses!

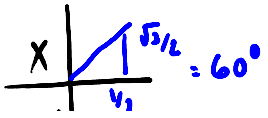
* LOOK AT HANDOUT OF INVERSE GRAPHS *

Notice that the x- and y-axis switch for the inverse.

$\sin \frac{\pi}{6} = \frac{1}{2}$ $\quad \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

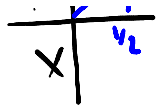
$\sin \theta = \frac{\sqrt{3}}{2} \longrightarrow \arcsin \frac{\sqrt{3}}{2} = \theta \quad \theta = \frac{\pi}{3}$

Sin: $\frac{y}{1}$



$= 60°$

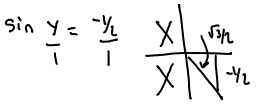
* for the restriction $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, inverse sine has to be



inverse sine has to be in the 1st or 4th quadrant.

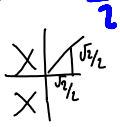
Ex. 1 Find the exact value.

a) $\arcsin(-1/2)$



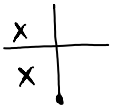
$\theta = -\frac{\pi}{6}$

b) $\sin^{-1}\frac{\sqrt{2}}{2}$



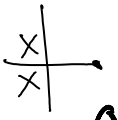
$\theta = \frac{\pi}{4}$

c) $\sin^{-1}(-1)$



$\theta = -\pi/2$

d) $\arcsin(0)$



$\theta = 0$ rads

Defn of Other Trig functions

$y = \arccos x$ $(y = \cos^{-1} x)$ if and only if $\cos y = x$ Domain $-1 \le x \le 1$ Range $0 \le y \le \pi$

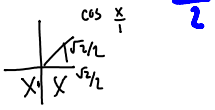
* To find the inverse of cosine, use quad I and II. arccos 

$y = \arctan x$ $(y = \tan^{-1} x)$ if and only if $\tan y = x$ Domain $-\infty < x < \infty$ Range $-\pi/2 < y < \pi/2$

To find the inverse of tangent, use quad. I or IV. arc tan 

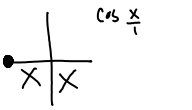
Ex. 2 Find the exact value

a) $\arccos\frac{\sqrt{2}}{2}$



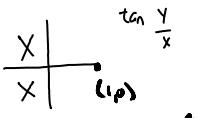
$\theta = \frac{\pi}{4}$

b) $\cos^{-1}(-1)$



$\theta = \pi$

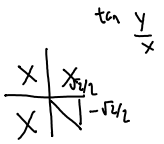
c) $\arctan(0)$



0°

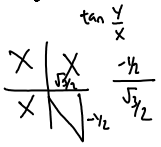
$\theta = 0$ rad

d) $\tan^{-1}(-1)$



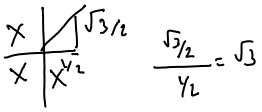
$\theta = -\frac{\pi}{4}$

e) $\tan^{-1}(-\sqrt{3}/3)$



$\theta = -\frac{\pi}{6}$

f) arc tan $(\sqrt{3})$

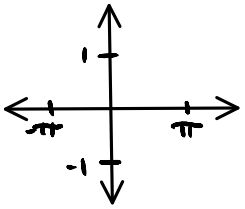


$\theta = \frac{\pi}{3}$

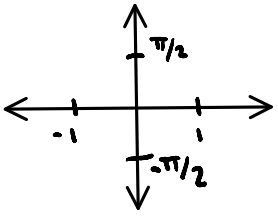
Continued

When graphing inverse functions, the axes change values. The values of x are now y and y are now x.

$y = \sin x$



$y = \arcsin x$



Ex.3 Use the calculator to evaluate

a) $\sin^{-1}(-.2386)$
$\approx -.2409$ rads

b) $\arccos(1.823)$
Undefined
* Remember that Domain is $-1 \le x \le 1$
larger than 1

c) $\tan^{-1}(6.893)$
≈ 1.4267 rads

d) $\arccos(.6832)$
≈ 0.8187 rads

e) $\arccos(3)$ ← reciprocal
$\arccos(1/3)$
≈ 1.2310 rads

$\sec = \frac{1}{\cos}$

$\csc = \frac{1}{\sin}$

$\cot = \frac{\cos}{\sin}$

Inverse Functions
2nd sin → $\sin^{-1}(\ )$
" " cos → $\cos^{-1}(\ )$
" " tan → $\tan^{-1}(\ )$

Compositions of Function

IF $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$

$\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$

If $-1 \le x \le 1$ and $0 \le y \le \pi$

$\cos(\arccos x) = x$ and $\arccos(\cos y) = y$

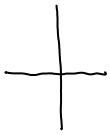
If x is a real # and $-\frac{\pi}{2} < y < \frac{\pi}{2}$

$\tan(\arctan x) = x$ and $\arctan(\tan y) = y$

Ex.4 Find the exact value

a) $\cos(\tan^{-1}(1))$

$\tan^{-1}(1)$



b) $\sec(\sin^{-1}(1/2))$

c) $\sin(\tan^{-1}(1/2))$

d) $\tan(\arccos(-1/3))$

$(-1)^2 + b^2 = (3)^2$

$b^2 = 8$

$b = 2\sqrt{2}$

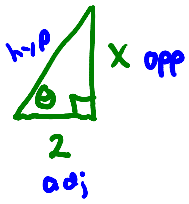
$\tan \frac{y}{x}$

$\frac{2\sqrt{2}}{-1} = -2\sqrt{2}$

$2 \cdot 2 \cdot \sqrt{3}$

$4\sqrt{3}$

e) Use an "Inverse function" to write θ as a function of x.



$\tan\theta = \frac{x}{2} \longrightarrow \tan^{-1}\frac{x}{2} = \theta$

f) $\sin(\sin^{-1}(1/2))$

$1/2$

g) $\sin^{-1}(\sin(\pi/6))$

$\pi/6$

h) $\sec(\arcsin(-5/7))$

$\frac{7\sqrt{6}}{12}$

Hw 4.7 Tb pg. 351-352 #'s 4-7 all
8-14 even
18-20 even
26-44 even
70, 71
} 22 problems