4.5 Graphs of Sine and Cosine

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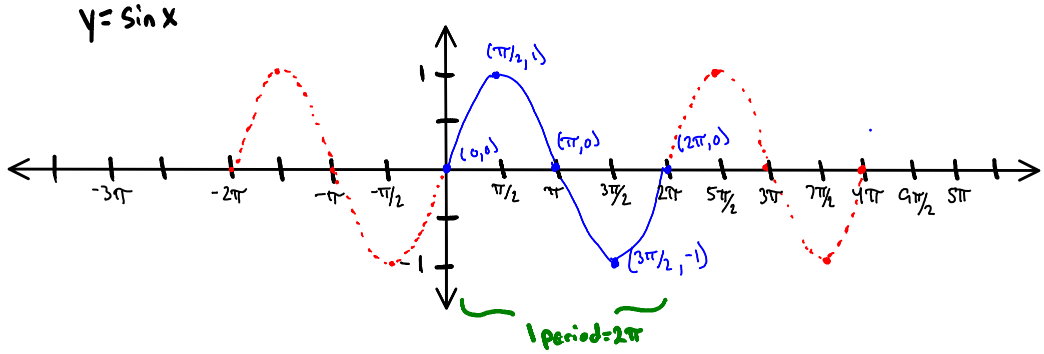
Trig functions are **Periodic Functions**, this means there is a basic shape that repeats itself after a fixed **Period** of time.

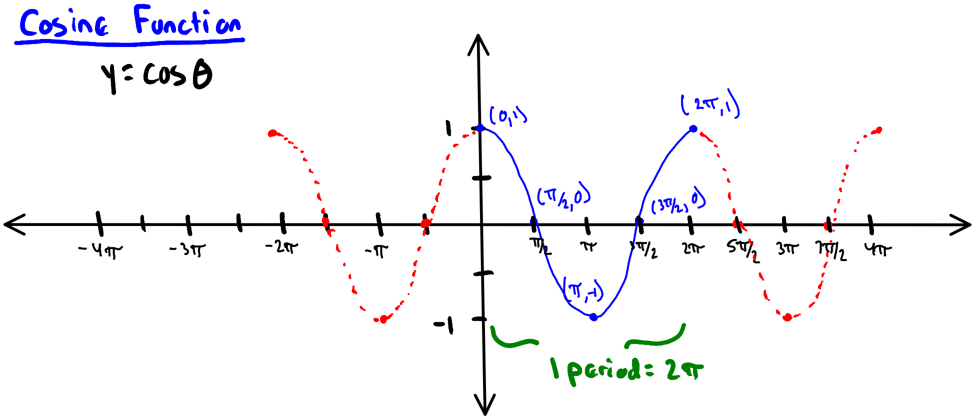
Sine Function

$y = \sin x$



Cosine Function

$y = \cos\theta$



Sin and cos functions share many similar key items:

	$f(x) = \sin(x)$	$f(x) = \cos(x)$
Domain	$\mathbb{R}$	$\mathbb{R}$
Range	$-1 \le y \le 1$	$-1 \le y \le 1$
Period	$[0, 2\pi]$ 2π	$[0, 2\pi]$
Key Points	$(0,0)$ $(\pi/2, 1)$ $(\pi, 0)$ $(\frac{3\pi}{2}, -1)$ $(2\pi, 0)$ Inter. Max Inter Min Inter. Max → Quarter Period; Inter → Half Period; Min → Three Quarter Period; Inter. → Full Period	$(0,1)$ $(\pi/2, 0)$ $(\pi, -1)$ $(3\pi/2, 0)$ $(2\pi, 1)$ Max Inter Min Inter Max
Symmetry	Origin	y-axis
Even / odd	odd	even

Period of a trig function is the distance from $x=0$ it takes to graph the basic shape (no repeat).

Domain of a trig function is the ∡ measure of θ.
Range of a trig function is the value of the trig function at a certain ∡ measure.

ex: $\sin 30° = 1/2$ — Domain $30°$ — Range $1/2$

$\sin \frac{\pi}{6} = \frac{1}{2}$ — Domain $\pi/6$ — Range $1/2$

Since trig functions are periodic, you can use the period to find an equivalent ∡ measure on the unit circle.

$$\sin\theta = \sin(\theta \pm 360n) \quad (\theta \pm 2\pi n)$$

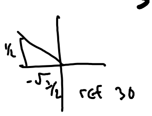
$$\cos\theta = \cos(\theta \pm 360n) \quad (\theta \pm 2\pi n)$$

Ex. 1 Use the period of the trig function to change ∡ measure to an equivalent ∡ measure on the unit circle.

a) $\sin 510°$

$\sin(510° - 360) =$

$\sin(150°) = \frac{1}{2}$

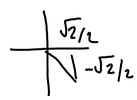


b) $\cos \frac{15\pi}{4}$

$\cos\left(\frac{15\pi}{4} - 2\pi\right) =$

$\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\left(\frac{15}{4} - \frac{8}{4}\right)$



Recall

$$y = a|x \pm b| \pm c$$

+b moves left
-b moves right

+c moves up
-c moves down

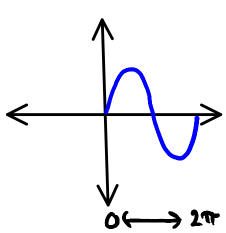
$a > 1$ vert. stretch
$0 < a < 1$ vert. compr.

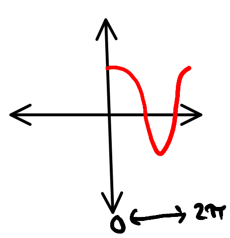
$a > 0$ above x-axis
$a < 0$ below x-axis

Sine — $y = \sin x$ ⟵ Parent Graphs ⟶ Cosine — $y = \cos x$

$$y = \pm a \sin(bx \pm c) \pm d$$

$$y = \pm a \cos(bx \pm c) \pm d$$





Transformations of Sine and Cosine Graphs

Vertical Transformations "$\pm d$"

moves the graph up $+d$, moves the graph down $-d$

Ex. 2 Describe the transformation

a) $y = \sin(x) + 5$

shifts 5 units up, all points

b) $y = \cos(x) - 2$

shifts 2 units down, all points

Reflection over the x-axis $a < 0$

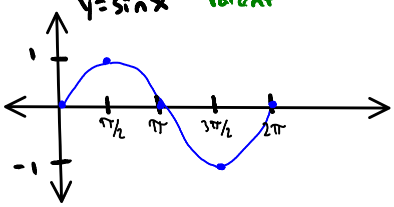
$y = -a\sin(bx \pm c) \pm d$ $\quad$ $y = -a\cos(bx \pm c) \pm d$

Ex. 3 Graph the function.

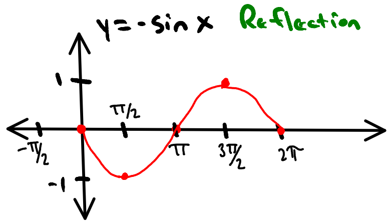
* Always graph the parent graph first! Label all points on the final graph!

a) $y = -\sin(x) - 2$; $0 \le x \le 2\pi$

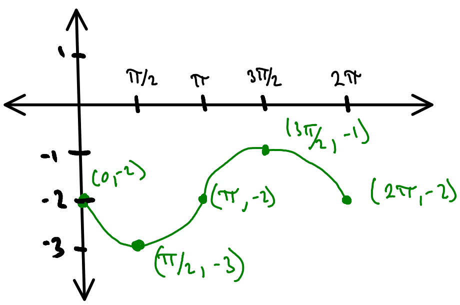
$y = \sin x$ Parent



$y = -\sin x$ Reflection

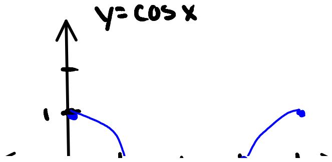


$y = -\sin(x) - 2$

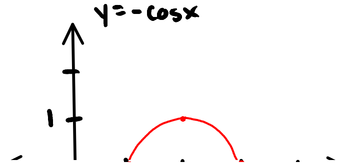


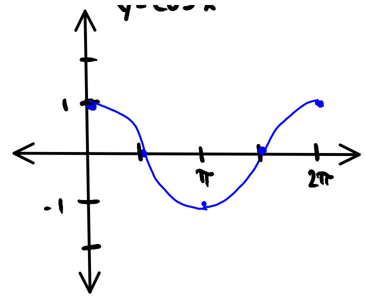
b) $y = -\cos(x) - 2$; $0 \le x \le 2\pi$

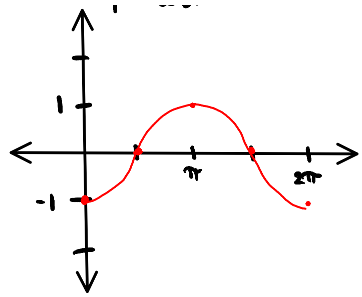
$y = \cos x$

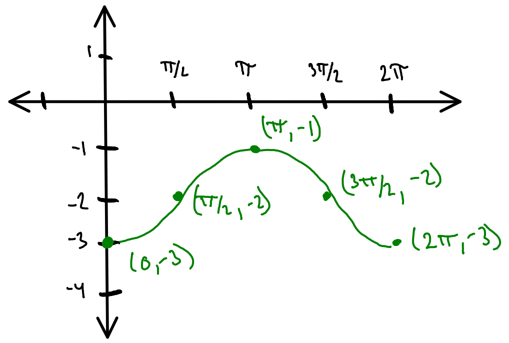


$y = -\cos x$

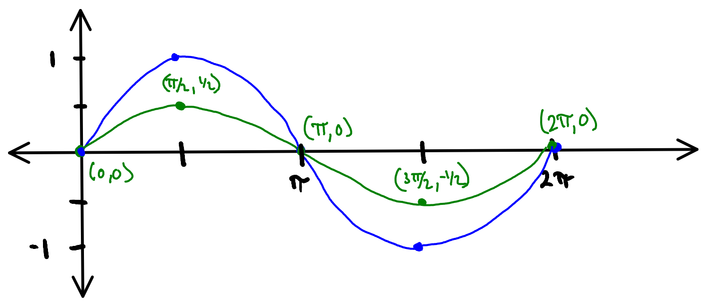






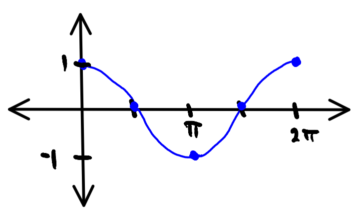


c) $y = \frac{1}{2}\sin x$; $0 \le x \le 2\pi$ * multiply all y-values by 1/2!

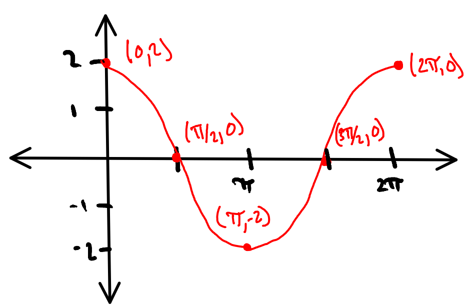


	y-values
(0, 0)	$0 \cdot \frac{1}{2} = 0$
(π/2, 1)	$1 \cdot \frac{1}{2} = \frac{1}{2}$
(π, 0)	$0 \cdot \frac{1}{2} = 0$
(3π/2, -1)	$-1 \cdot \frac{1}{2} = -\frac{1}{2}$
(2π, 0)	$0 \cdot \frac{1}{2} = 0$

d) $y = 2\cos x$; $0 \le x \le 2\pi$



	y-values
(0, 1) →	2
(π/2, 0) →	0
(π, -1) →	-2
(3π/2, 0) →	0
(2π, 1) →	2



Amplitude (It is the height of the wave)

Represents half the distance between the maximum and minimum values of the function.

Given $y = a\sin(bx \pm c) \pm d$, then the Amplitude $= |a|$

a acts as a scaling factor. it causes vertical stretch $(a>1)$ and vertical compression $(0<a<1)$

$\text{Amp} = \frac{1}{2}|\max - \min|$ ← Use when a graph is given.

* Changing "a" results in a different range!

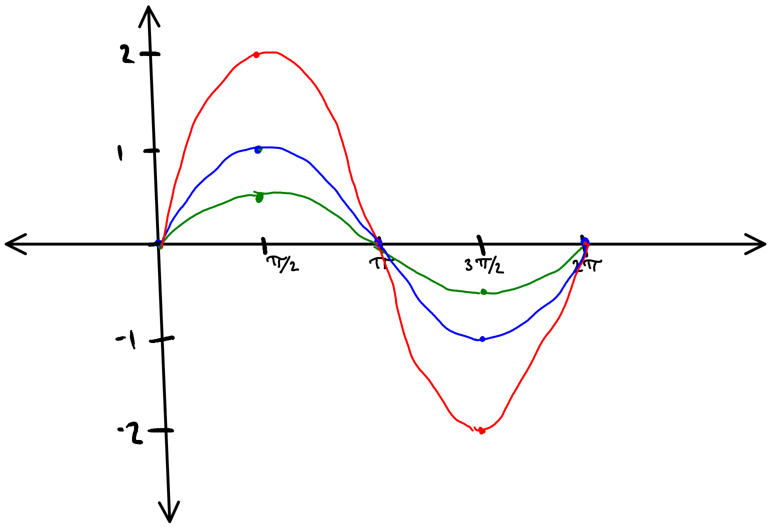
Ex.4 Graph all functions on the same graph. List domain and range for each.

$0 \le x \le 2\pi$

a) $y = \sin x$

b) $y = \frac{1}{2}\sin x$

c) $y = 2\sin x$



Ex.5 State the amplitude of each function.

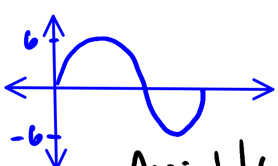
a) $y = 5\sin x$

$\text{Amp} = |5|$
$= 5$

b) $y = -3\cos x$

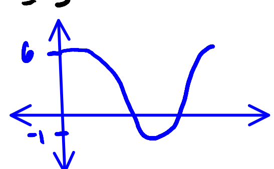
$\text{Amp} = |-3|$
$= 3$

c)



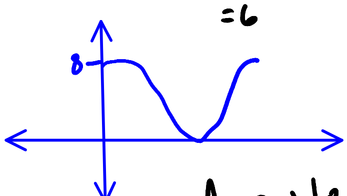
$\text{Amp} = \frac{1}{2}|6-(-6)|$
$= \frac{1}{2}|12|$
$= 6$

d)



$\text{Amp} = \frac{1}{2}|6-(-1)|$
$= \frac{1}{2}|7|$
$= 3.5$

e)



$\text{Amp} = \frac{1}{2}|8-0|$
$= \frac{1}{2}|8|$
$= 4$

Period of Sine and Cosine Functions

Let b be a positive real #, then the period of
$y = a\sin(bx)$ and $y = a\cos(bx)$ is given by

$$\text{Period} = \frac{2\pi}{b}$$

$y = \sin x$
$b = 1$ per $= \frac{2\pi}{1} = 2\pi$

* Horizontal stretch and compression comes from changing the period!

$b > 1$ horizontal compression
$0 < b < 1$ horizontal stretch

Ex. 6 Find the period for each function and state if it is horizontal compression or stretch.

a) $y = \cos x$

per $= \frac{2\pi}{b}$

$= \frac{2\pi}{1}$

$= 2\pi$

b) $y = \sin 4x$

per $= \frac{2\pi}{4}$; $b = 4$

$= \frac{2\pi}{4}$

$= \frac{\pi}{2}$

horizontal Compression

c) $y = \cos(\frac{1}{3}x)$

6π

Ex. 7 Graph the Function

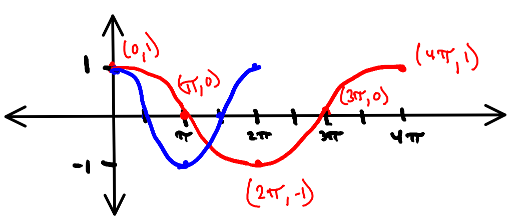
a) $y = \cos(x/2)$

$b = \frac{1}{2}$; horizontal stretch

per: $\frac{2\pi}{1/2} \rightarrow 2\pi \cdot \frac{2}{1} = 4\pi$ ← New Period

$\frac{4\pi}{4} = \pi$

$[0, \pi, 2\pi, 3\pi, 4\pi]$



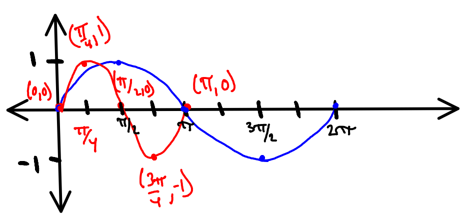
b) $y = \sin 2x$

$b = 2$, horizontal compression

period $= \frac{2\pi}{2}$

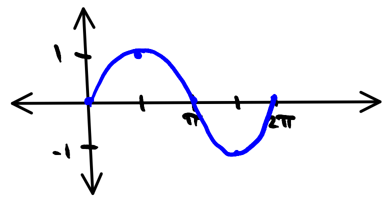
$= \pi$

$\frac{\pi}{4} = [0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi]$

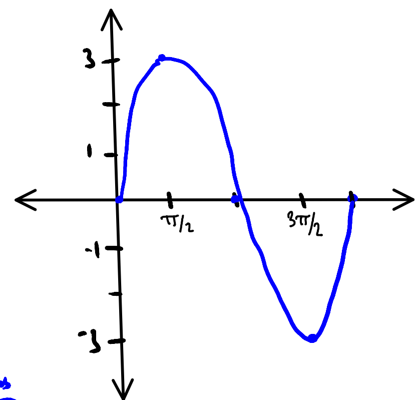


$\frac{\pi}{4} = [0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi]$

c) $y = 3\sin(2x)$ $\quad 0 \le x \le 2\pi$



Amp $= |3|$
$= 3$



$b = 2$; horizontal compress

per $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$\frac{\pi}{4} = [0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi]$