4.5 Graphs of Sine and Cosine

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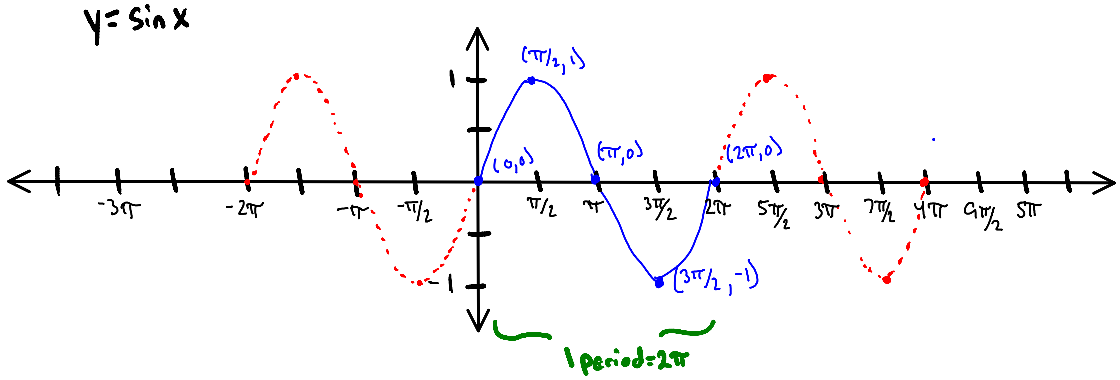
12:30 PM

Trig functions are **Periodic Functions**, this means there is a basic shape that repeats itself after a fixed **Period** of time.

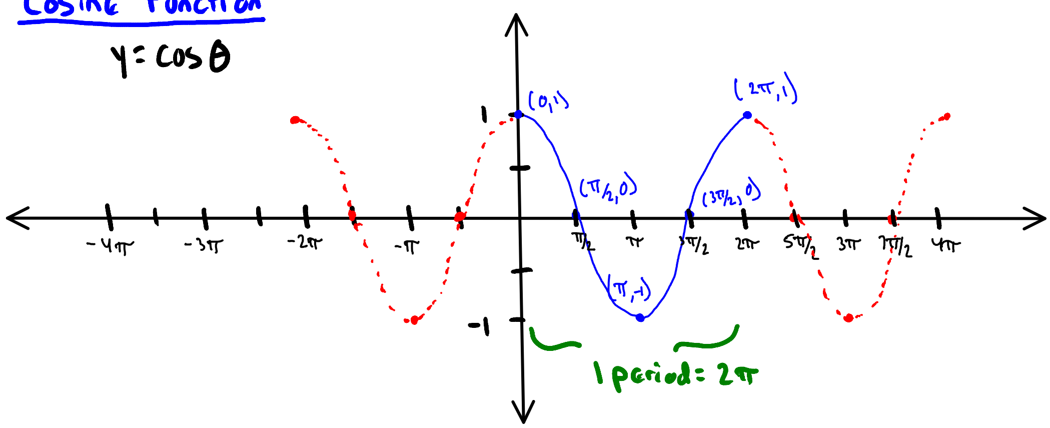
Sine Function

$y = \sin x$



Cosine Function

$y = \cos\theta$



Sin and cos functions share many similar key items.

	$f(x) = \sin(x)$		$f(x) = \cos(x)$
Domain	$\mathbb{R}$		$\mathbb{R}$
Range	$-1 \le y \le 1$		$-1 \le y \le 1$
Period	$[0, 2\pi]$	2π	$[0, 2\pi]$
Key Points	$(0,0)$ $(\pi/2, 1)$ $(\pi, 0)$ $(\frac{3\pi}{2}, -1)$ $(2\pi, 0)$		$(0,1)$ $(\pi/2, 0)$ $(\pi, -1)$ $(3\pi/2, 0)$ $(2\pi, 1)$
	Inter. Max Inter Min Inter.		Max Inter Min Inter Max
	Quarter Period, Half Period, Three Quarter Period, Full Period		
Symmetry	Origin		y-axis
Even / odd	odd		even

Period of a trig function is the distance from $x=0$ it takes to graph the basic shape (no repeat).

Domain of a trig function is the ∡ measure of θ.
Range of a trig function is the value of the trig function at a certain ∡ measure.

ex: $\sin 30° = 1/2$
Domain $30°$
Range $1/2$

$\sin \frac{\pi}{6} = \frac{1}{2}$
Domain $\pi/6$
Range $1/2$

Since trig functions are periodic, you can use the period to find an equivalent ∡ measure on the unit circle.

$$\sin\theta = \sin(\theta \pm 360n)$$
$$(\theta \pm 2\pi n)$$

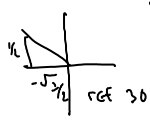
$$\cos\theta = \cos(\theta \pm 360n)$$
$$(\theta \pm 2\pi n)$$

__Ex. 1__ Use the period of the trig function to change ∡ measure to an equivalent ∡ measure on the unit circle.

a) $\sin 510°$

$\sin(510° - 360) =$

$\sin(150°) = \frac{1}{2}$

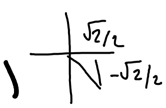


b) $\cos \frac{15\pi}{4}$

$\cos\left(\frac{15\pi}{4} - 2\pi\right) =$

$\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\left(\frac{15}{4} - \frac{8}{4}\right)$



Recall

$$y = a|x \pm b| \pm c$$

+b moves left
-b moves right

+c moves up
-c moves down

$a > 1$ vert. stretch
$0 < a < 1$ vert. compr.

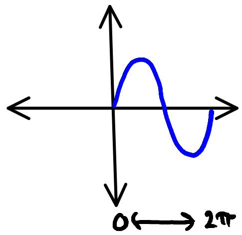
$a > 0$ above x-axis
$a < 0$ below x-axis

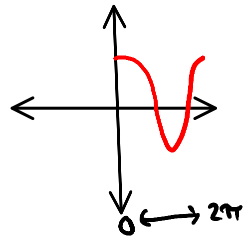
__Sine__ __Cosine__

$y = \sin x$ ⟵ Parent Graphs ⟶ $y = \cos x$

$y = \pm a \sin(bx \pm c) \pm d$

$y = \pm a \cos(bx \pm c) \pm d$





Transformation of Sine and Cosine Graphs

Vertical Transformations "$\pm d$"

moves the graph up $+d$, moves the graph down $-d$

Ex. 2 Describe the transformation

a) $y = \sin(x) + 5$

shifts 5 units up, all points

b) $y = \cos(x) - 2$

shifts 2 units down, all points

Reflection over the x-axis $a < 0$

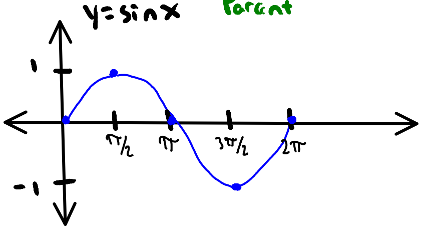
$y = -a\sin(bx \pm c) \pm d$ $\quad$ $y = -a\cos(bx \pm c) \pm d$

Ex. 3 Graph the function.

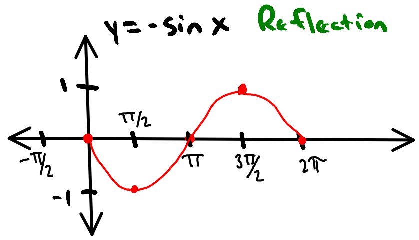
* Always graph the parent graph first! Label all points on the final graph!

a) $y = -\sin(x) - 2$; $0 \le x \le 2\pi$

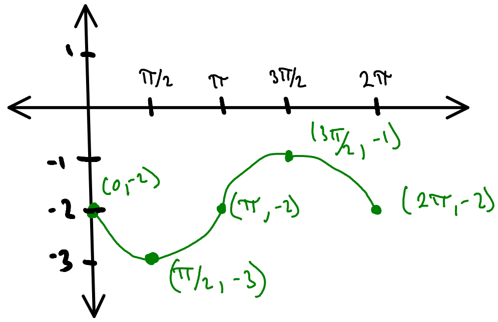
$y = \sin x$ Parent



$y = -\sin x$ Reflection

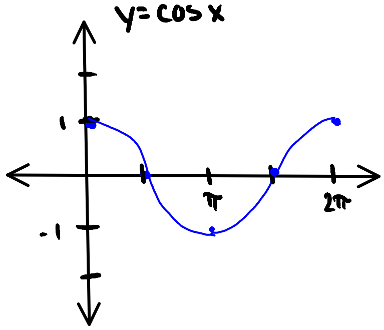


$y = -\sin(x) - 2$

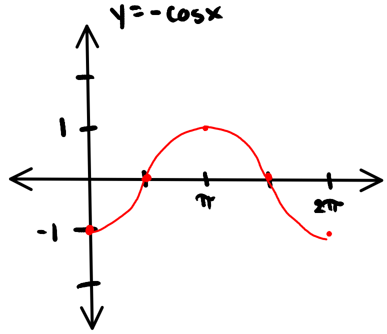


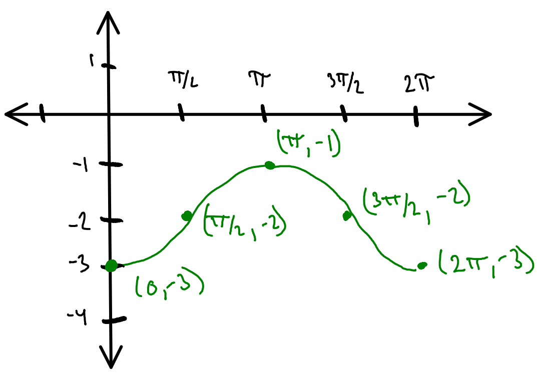
b) $y = -\cos(x) - 2 \; ; \; 0 \le x \le 2\pi$

$y = \cos x$



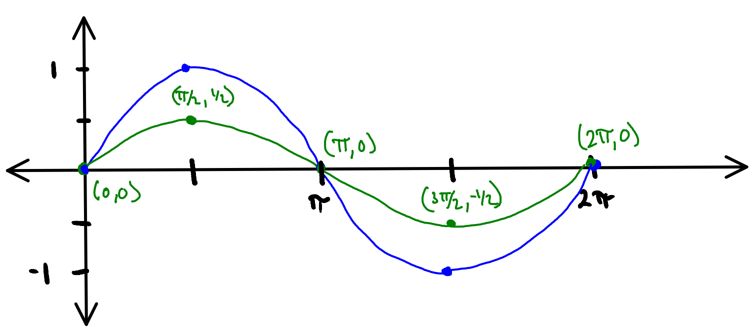
$y = -\cos x$





c) $y = \frac{1}{2}\sin x \; ; \; 0 \le x \le 2\pi$

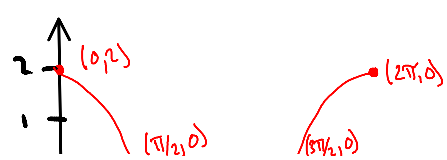
* multiply all y-values by ½!

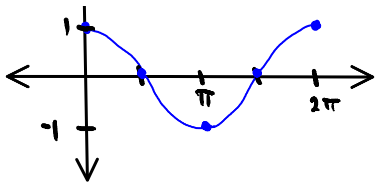


	y-values
$(0,0)$	$0 \cdot \frac{1}{2} = 0$
$(\frac{\pi}{2}, 1)$	$1 \cdot \frac{1}{2} = \frac{1}{2}$
$(\pi, 0)$	$0 \cdot \frac{1}{2} = 0$
$(\frac{3\pi}{2}, -1)$	$-1 \cdot \frac{1}{2} = -\frac{1}{2}$
$(2\pi, 0)$	$0 \cdot \frac{1}{2} = 0$

d) $y = 2\cos x \; ; \; 0 \le x \le 2\pi$

y-values





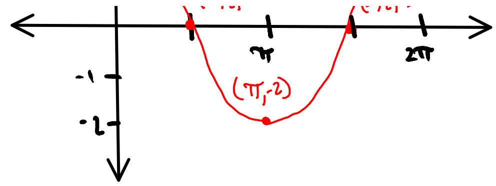
$(0,1) \to 2$

$(\pi/2,0) \to 0$

$(\pi,-1) \to -2$

$(3\pi/2,0) \to 0$

$(2\pi,1) \to 2$



Amplitude (It is the height of the wave)

Represents half the distance between the maximum and minimum values of the function.