4.4 Trig. Functions of Any Angle

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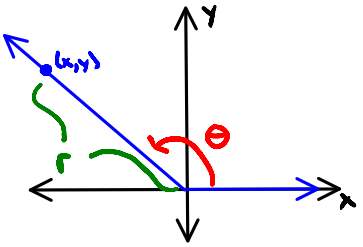
10:58 AM

Defn. of Trigometric Functions

Let θ be an angle in standard position with (x,y) a point on the terminal side of θ and

$r = \sqrt{x^2+y^2} \neq 0$

$x^2+y^2=r^2$ eqn circle



$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

* If $x=0$, then tan and sec are undefined.
If $y=0$, the csc and cot are undefined.

Ex. 1

Let $(5,12)$ be on the terminal side of θ. Determine sin, cos, and tan θ.

$r = \sqrt{x^2+y^2}$

$r = \sqrt{(5)^2+(12)^2}$

$r = \sqrt{169}$

$r = 13$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{12}{13}$ $\cos\theta = \frac{5}{13}$ $\tan\theta = \frac{12}{5}$

Ex. 2

What is $\sin\theta$ when $\cos\theta = \frac{8}{17}$ and the terminal side of θ is in Quad. IV?

$17 = \sqrt{(8)^2+y^2}$

$(17)^2 = \left(\sqrt{64+y^2}\right)^2$

$289 = 64+y^2$

$225 = y^2$

$y = \pm 15$

$\sin\theta = \frac{-15}{17}$

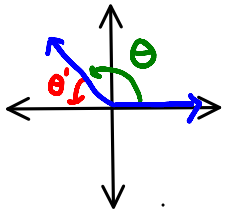
Ex. 3

Given $\cot\theta = \frac{-4}{5}$ and $\cos\theta < 0$, what is $\sin\theta$ and $\sec\theta$?

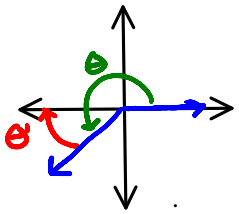
O | x x | x	$r = \sqrt{x^2+y^2}$ $r = \sqrt{(-4)^2+(5)^2}$ $r = \sqrt{41}$	$\sin\theta = \frac{y}{r}$ $\sin\theta = \frac{5}{\sqrt{41}} \rightarrow \frac{5\sqrt{41}}{41}$	$\sec\theta = \frac{r}{x}$ $\sec\theta = \frac{\sqrt{41}}{-4}$

Let θ be an $\angle$ in standard position. Its reference $\angle$ is the acute $\angle$ θ' formed by the terminal side of θ and the horizontal axis.

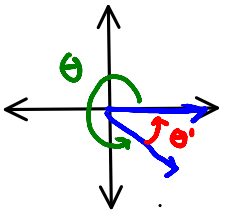
* Don't need ref. $\angle$ for Quad I *



ref $\theta' \begin{cases} = \pi - \theta \\ = 180 - \theta \end{cases}$



ref $\theta' \begin{cases} = \theta - \pi \\ = \theta - 180 \end{cases}$



ref $\theta' \begin{cases} = 2\pi - \theta \\ = 360 - \theta \end{cases}$

Ex. 4 Determine the ref. $\angle$ for the following:

a) 120°

$180 - 120 = 60^\circ$

$\theta' = 60^\circ$

b) 330°

$360 - 330 = 30^\circ$

ref $\angle = 30^\circ$

c) 225°

$225 - 180 = 45^\circ$

ref $\angle = 45^\circ$

d) -200°

$-200 + 360 = 160$

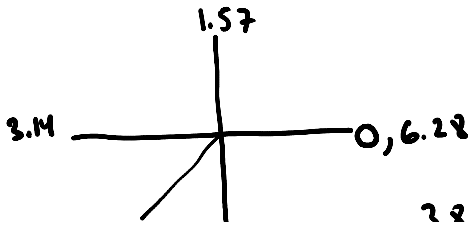
$180 - 160 = 20^\circ$

ref $\angle$ 20°

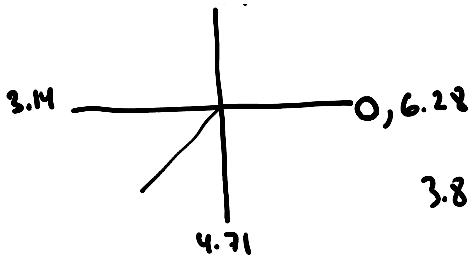
e) $\frac{2\pi}{3}$

f) $\frac{11\pi}{6}$

g) $\theta = 3.8$ rads



$3.8 - 3.14 = .66$



$3.8 - 3.14 = .66$ rads