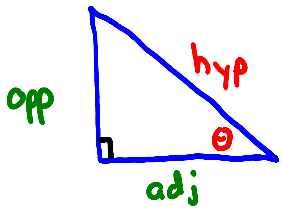
4.3 Right Triangle Trig

Monday, March 16, 2015
10:53 AM



θ is an acute ∠ of a Rt △.

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

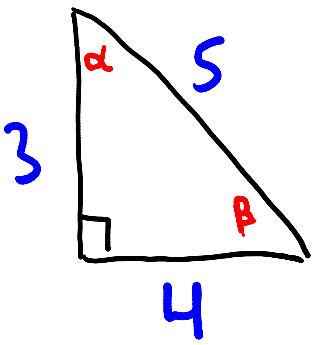
$\csc\theta = \frac{hyp}{opp}$ $\sec\theta = \frac{hyp}{adj}$ $\cot\theta = \frac{adj}{opp}$

Reciprocals of top 3 functions

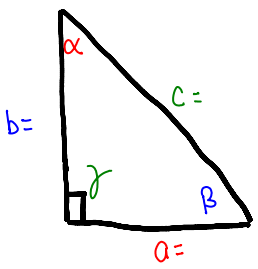
You can use Rt △ trig to solve for any missing sides or ∠ measures

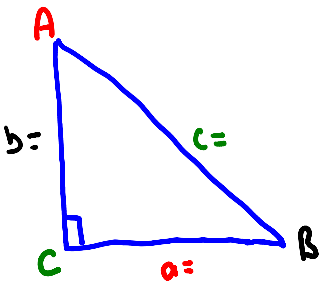
* If not stated, the side lengths you should round to the hundredths and the ∠ measure round to the tenths.

Ex. 1 Write the trig ratios for ∠α and ∠β

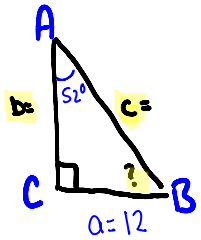


α	β
$\sin\alpha = 4/5$	$\sin\beta = 3/5$
$\cos\alpha = 3/5$	$\cos\beta = 4/5$
$\tan\alpha = 4/3$	$\tan\beta = 3/4$
$\csc\alpha = 5/4$	$\csc\beta = 5/3$
$\sec\alpha = 5/3$	$\sec\beta = 5/4$
$\cot\alpha = 3/4$	$\cot\beta = 4/3$





Ex. 2 Solve the right $\triangle ABC$ when: $m\angle A = 52°$, $\overline{CB} = 12$ and $\angle C$ is the right angle.



$m\angle B$

$90° - 52°$

$38°$

* You must make sure you are in the correct mode!

* Use the info given to solve!

Side c

$\sin 52° = \frac{12}{c}$

$c \sin 52° = 12$

$c = \frac{12}{\sin 52°}$

$c \approx 15.23$

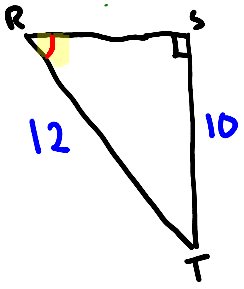
Side b

$\tan 52° = \frac{12}{b}$

$b = \frac{12}{\tan 52°}$

$b \approx 9.38$

Ex. 3 In $\triangle RST$, determine the measure of $\angle R$.



$m\angle R$

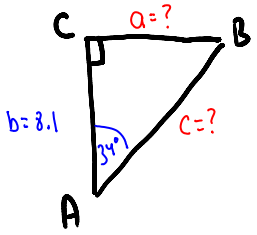
$\sin R = \frac{10}{12}$

2nd Sin $\rightarrow \sin^{-1}(10/12)$

$R \approx 56.4°$

* To find any $\angle$ measure press 2nd "Trig function"

Ex. 4 Solve for a, c, and $m\angle B$ if $m\angle A$ is 34° and b = 8.1 in



$m\angle B$

$90° - 34° = 56°$

Side c

$\cos 34 = \frac{8.1}{c}$

$c \approx 9.77$ in

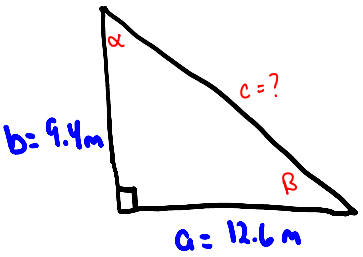
Side a

$\tan 34 = \frac{a}{8.1}$

$a \approx 5.46$ in

Ex. 5 Solve the Rt $\triangle$.

Side c



$(12.6)^2 + (9.4)^2 = c^2$

$c^2 = 247.12$

$c \approx 15.72\text{ m}$

m∠α

$\tan\alpha = \frac{12.6}{9.4}$

$\tan^{-1}(12.6/9.4)$

$\alpha = 53.3^\circ$

m∠β

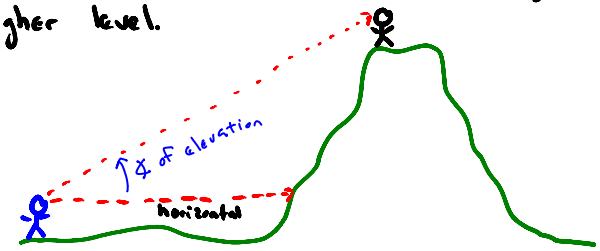
$\tan\beta = \frac{9.4}{12.6}$

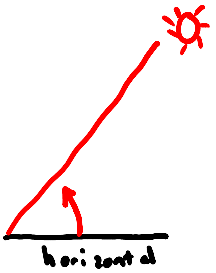
$\tan^{-1}(9.4/12.6)$

$\beta \approx 36.7^\circ$

Angle of Elevation

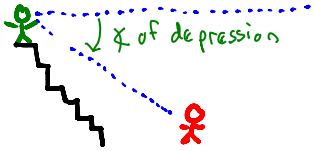
Is the angle between a horizontal line and the line of sight of an observer to an object at a higher level.



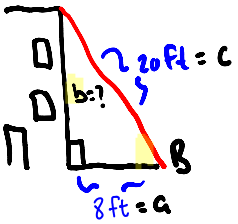


Angle of Depression

The angle between a horizontal line and the line of sight of an observer to an object at a lower level.



Ex. 6



Side b

$(8)^2 + b^2 = (20)^2$

$b^2 = 336$

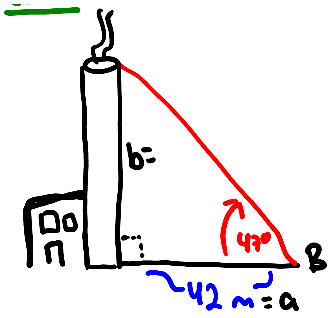
$b \approx 18.33\text{ ft}$

m∠B

$\cos B = \frac{8}{20}$

$B = 66.4^\circ$

Ex. 7

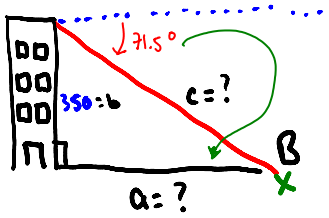


Side b

$\tan 47^\circ = \frac{b}{42}$

$b \approx 45.04$ m

Ex. 3



alternate interior angles

Side c

$\sin 71.5^\circ = \frac{350}{c}$

$c = \frac{350}{\sin 71.5^\circ}$

$c \approx 369.07$ ft

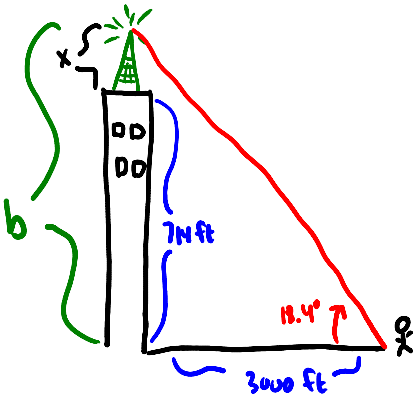
Side a

$\tan 71.5^\circ = \frac{350}{a}$

$a = \frac{350}{\tan 71.5}$

$a \approx 117.11$ ft

Ex. 9



$\tan 18.4 = \frac{b}{3000}$

$b = 3000 \tan 18.4$

$b \approx 997.97$ ft

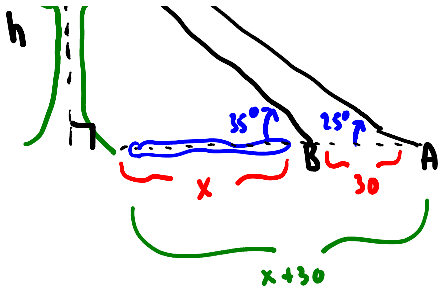
TV Tower

$997.97 - 714$

283.97 ft

Ex. 10

$\tan 35^\circ = \frac{h}{\quad}$ $\tan 25^\circ = \frac{h}{\quad}$



x $\quad$ $x+30$

$x\tan 35° = h$ $\quad$ $h = (x+30)\tan 25$

$x\tan 35 = x\tan 25 + 30\tan 25$

$-x\tan 25 \quad -x\tan 25$

$x\tan 35 - x\tan 25 = 30\tan 25$

$x(\tan 35 - \tan 25) = 30\tan 25$

$$x = \frac{30\tan 25}{(\tan 35 - \tan 25)}$$

$x \approx 59.91$ m

$59.91\tan 35 = h$

$h \approx 41.93$ m

4.3 Continued 3/18/15

Trig. Identities

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Even - Odd Identities

$\sin(-\theta) = -\sin\theta$ $\quad$ $\csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad$ $\sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad$ $\cot(-\theta) = -\cot\theta$

Domain → $\sin 45° = \frac{\sqrt{2}}{2}$ ← Range

$\sin\theta = y$

cos / sec are Even Functions — x sign change, but y sign does not

sin / csc, tan / cot } are Odd Functions — x sign change, and y sign change

ex

$\cos(30) = \frac{\sqrt{3}}{2}$ even $\cos(-30) = \frac{\sqrt{3}}{2}$

$\sin(30) = \frac{1}{2}$ odd $\sin(-30) = -\frac{1}{2}$

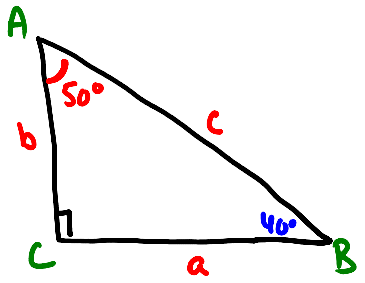
$\tan x = -\frac{3}{4}$ odd $\tan(-x) = \frac{3}{4}$

Cofunctions of Complementary Angles 90° or $\frac{\pi}{2}$

$\sin(90^\circ - \theta) = \cos\theta$
$\cos(90^\circ - \theta) = \sin\theta$
$\tan(90^\circ - \theta) = \cot\theta$

$\csc(90^\circ - \theta) = \sec\theta$
$\sec(90^\circ - \theta) = \csc\theta$
$\cot(90^\circ - \theta) = \tan\theta$

$\sin 40 = (40 - 40) = \cos 50$



Ex Write each function in terms of its cofunction.

$\sin 18^\circ = \cos\beta$
$\sin 18^\circ = \cos 72^\circ$

$\cos\frac{\pi}{4} = \sin\beta$
$\cos\frac{\pi}{4} = \sin\frac{\pi}{4}$

$\cot\frac{\pi}{3} = \tan\beta$
$\cot\frac{\pi}{3} = \tan\frac{\pi}{6}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin^2\theta = (\sin\theta)^2$ it is not $\sin\theta^2$

$\sin^2(30) = (\sin 30)^2$
$= (\frac{1}{2})^2$
$= \frac{1}{4}$

~~$\sin^2(30)$~~
~~$\sin(30)^2$~~
~~$\sin 900$~~

Proof

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$ $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}$

$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$1 + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta} \rightarrow 1 + \cot^2\theta = \csc^2\theta$

$\frac{1}{\sin^2\theta} = \csc^2\theta$

Ex.11 Find the remaining trig functions using trig IDs.
(Do not use right Δs)

a) $\sec\theta = 4$, $\sin\theta < 0$

$\sec\theta = 4$	$\sin\theta = -\frac{\sqrt{15}}{4}$	$\cos\theta = \frac{1}{4}$	$\tan\theta = -\sqrt{15}$
$\cos\theta = \frac{1}{\sec\theta}$ (Reciprocal)	$\csc\theta = -\frac{4\sqrt{15}}{15}$	—	$\cot\theta = -\frac{\sqrt{15}}{15}$

Pythagorean ID
$1 + \tan^2\theta = \sec^2\theta$
$1 + \tan^2\theta = (4)^2$
$1 + \tan^2\theta = 16$
$\tan^2\theta = 15$
$\tan\theta = \pm\sqrt{15}$ Pick − b/c sec is in IV Quad
$\tan\theta = -\sqrt{15}$

$\cot = \frac{1}{\tan}$ Reciprocal
$\cot = \frac{1}{-\sqrt{15}} = -\frac{\sqrt{15}}{15}$

Quotient
$\tan\theta = \frac{\sin\theta}{\cos\theta}$
$-\sqrt{15} = \frac{\sin\theta}{1/4}$
$\sin\theta = -\frac{\sqrt{15}}{4}$

Reciprocal
$\csc\theta = \frac{1}{\sin\theta}$
$\csc\theta = \frac{1}{-\sqrt{15}/4} = -\frac{4\sqrt{15}}{15}$

Ex.12 Use trig. Ids to transform one-side of the eqn. into the other.

① ONLY CHANGE 1 SIDE
② CHANGE THE MORE COMPLICATED SIDE.

a) $\cos\theta \cdot \sec\theta = 1$ — Reciprocal is $\frac{1}{\cos\theta}$

$\cos\theta \cdot \frac{1}{\cos\theta} = 1$

$\frac{\cos\theta}{\cos\theta} = 1$

$1 = 1$

b) $\sin 30 \cdot \csc 30 = 1$ — Reciprocal Id of $\csc\theta$: $\frac{1}{\sin\theta}$

$\sin 30 \cdot \frac{1}{\sin 30} = 1$

$\frac{\sin 30}{\sin 30} = 1$

$1 = 1$

c) $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

Pythagorean: $\sec^2\theta - \tan^2\theta = 1$

$1 + \tan^2\theta - \tan^2\theta = 1$

$1 = 1$

d) $\csc\theta \tan\theta = \sec\theta$ — Quotient Id of $\tan\theta$

$\csc\theta\left(\frac{\sin\theta}{\cos\theta}\right) = \sec\theta$

Reciprocal Id of $\csc\theta$

$\frac{1}{\sin\theta}\left(\frac{\sin\theta}{\cos\theta}\right) = \sec\theta$

$\frac{1}{\cos\theta} = \sec\theta$

e) $\tan\theta\cos\theta = \sin\theta$

$\frac{\sin\theta}{\cos\theta}(\cos\theta) = \sin\theta$

$\sin\theta = \sin\theta$

f) $(1+\cos\theta)(1-\cos\theta) = \sin^2\theta$

$1 - \cos^2\theta = \sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = \sin^2\theta$

g) $\frac{\tan\theta + \cot\theta}{\tan\theta} = \csc^2\theta$

$$\frac{\tan\theta}{\tan\theta} + \frac{\cot\theta}{\tan\theta} = \csc^2\theta$$

$$1 + \frac{\cot\theta}{\tan\theta} = \csc^2\theta$$

$$1 + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta}} = \csc^2\theta \quad \rightarrow \quad \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$$