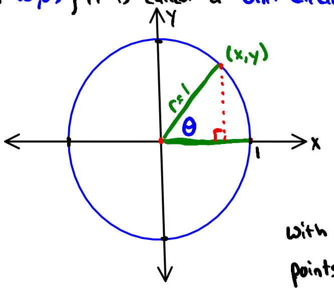
4.2 Unit Circle

Wednesday, March 11, 2015
10:23 AM

When a circle has a radius of 1 and the center is at the origin (0,0), it is called a Unit Circle.



$x^2+y^2=1$ $\quad (x-h)^2+(y-k)^2=r^2$

center (0,0)

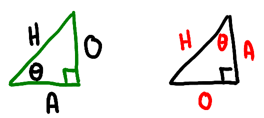
$r=1$

With the unit circle, you can find points on the circle using Rt Δ.

* Always bring a VERTICAL line down or up to the X-axis to form Rt Δ. *

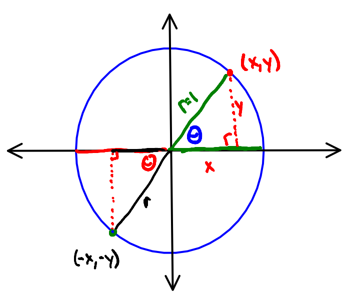
SOH	CAH	TOA
$\sin = \frac{O}{H}$	$\cos = \frac{A}{H}$	$\tan = \frac{O}{A}$
$\csc = \frac{H}{O}$	$\sec = \frac{H}{A}$	$\cot = \frac{A}{O}$

} Right Triangles



Six Trig Functions and the Unit Circle ; where $r=1$

Function	Unit circle Relation		
Sine	$\sin = \frac{y}{1}$	$\frac{y}{r}$	
Cosine	$\cos = \frac{x}{1}$	$\frac{x}{r}$	
Tangent	$\tan = \frac{y}{x}$	$\frac{\sin\theta}{\cos\theta}$	
Cosecant	$\csc = \frac{1}{y}$	$\frac{r}{y}$	$\frac{1}{\sin\theta}$
Secant	$\sec = \frac{1}{x}$	$\frac{r}{x}$	$\frac{1}{\cos\theta}$
Cotangent	$\cot = \frac{x}{y}$	$\frac{\cos\theta}{\sin\theta}$	



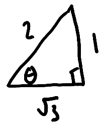
Notice that the y-coord is $\sin\theta$ and the x-coord is $\cos\theta$; only on the unit circle. $(x,y) \rightarrow (\cos\theta, \sin\theta)$

Review Special Right Triangles

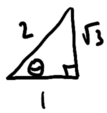
Cos θ ; only on the unit circle. $(x, y) \rightarrow (\cos\theta, \sin\theta)$

Review Special Right Triangles

θ: $30°$ or $\frac{\pi}{6}$



θ = $60°$ or $\frac{\pi}{3}$



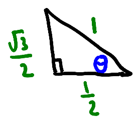
$\Big\}$ For $30°$-$60°$, hypotenuse is twice the short side. The middle leg is $\sqrt{3} \cdot$ short side

θ = $45°$ or $\frac{\pi}{4}$
Isosceles

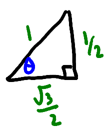
For $45°$, hypotenuse is $\sqrt{2} \cdot$ short side.

Special Right △s on the unit Circle (Reference triangles)

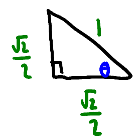
θ = $60°$ or $\frac{\pi}{3}$



θ = $30°$ or $\frac{\pi}{6}$



θ = $45°$ or $\frac{\pi}{4}$

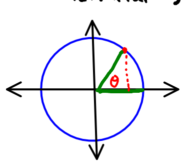


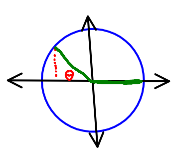
Study Filled out Unit Circle

When $x=0$ tangent and secant functions are undefined.
Happens at $90°$ or $\frac{\pi}{2}$ and $270°$ or $\frac{3\pi}{2}$

When $y=0$ cotangent and cosecant functions are undefined.
Happens at $0°$ or 0 rad, $180°$ or π, $360°$ or 2π

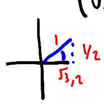
Reference Angle is an acute angle formed by the terminal side and x-axis



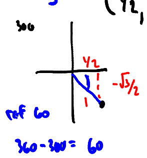


Ex. 1 What ordered pair on the Unit ⊙ corresponds to the following angle.

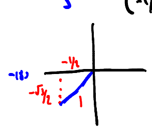
a) $\frac{\pi}{6}$
$30°$
$(\sqrt{3}/2, 1/2)$



b) $\frac{5\pi}{3}$ $(1/2, -\sqrt{3}/2)$
300

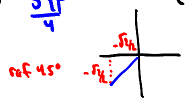


c) $-\frac{2\pi}{3}$ $(-1/2, -\sqrt{3}/2)$

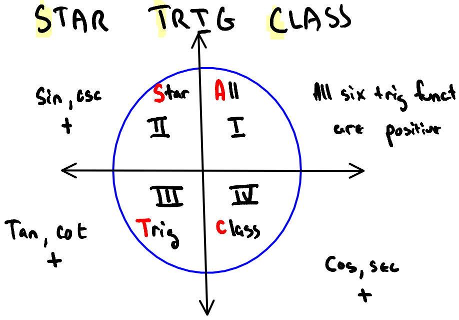


360-300 = 60

d) $\frac{5\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

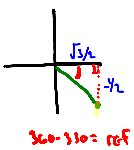


ALL STAR TRIG CLASS



Ex. 2 Give exact values

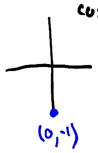
a) $\sin 330° = -\frac{1}{2}$



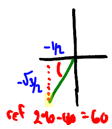
$\sin 330 = \frac{y}{1}$

b) $\cos 270° = 0$

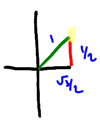
$\cos\theta = \frac{x}{1}$



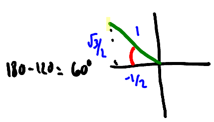
c) $\sin 240 = -\frac{\sqrt{3}}{2}$



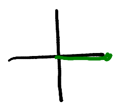
d) $\cos 30° = \frac{\sqrt{3}}{2}$



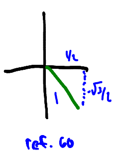
e) $\cos 120° = -\frac{1}{2}$



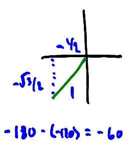
f) $\sin 0° = 0$



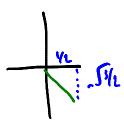
g) $\cos 300° = \frac{1}{2}$



h) $\sin -120° = -\frac{\sqrt{3}}{2}$

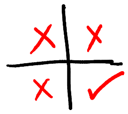


i) $\cos -60° = \frac{1}{2}$



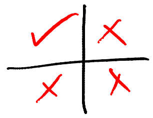
Ex. 3 What quadrant does θ lie?

a) $\sin\theta < 0$, $\cos\theta > 0$



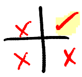
IV

b) $\tan\theta < 0$, $\cos\theta < 0$



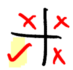
II

c) $\csc\theta > 0$, $\sec\theta > 0$



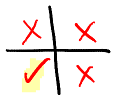
I

d) $\sec\theta < 0$, $\sin\theta < 0$



III

e) $\cot\theta > 0$, $\sin\theta < 0$



III

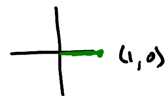
Ex. 4 Evaluate for each of the six trig. functions

a) $\theta = 0°$

$\sin\theta = 0$ $\quad \frac{y}{1}$

$\cos\theta = 1$ $\quad \frac{x}{1}$

$\tan\theta = 0$ $\quad \frac{y}{x}$ $\quad \frac{0}{1} = 0$



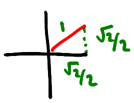
$\csc\theta =$ undefined $\quad \frac{1}{0}$

$\sec\theta = 1$

$\cot\theta =$ undefined $\quad \frac{1}{0}$

Reciprocals of the first 3!

b) $\theta = 45°$



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

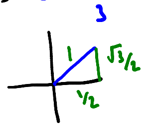
$\sin 45° = \frac{\sqrt{2}}{2}$ $\qquad \cos 45° = \frac{\sqrt{2}}{2}$ $\qquad \tan 45° = 1$ $\qquad \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$\csc 45° = \sqrt{2}$ $\qquad \sec 45° = \sqrt{2}$ $\qquad \cot 45° = 1$

$\frac{1}{\frac{\sqrt{2}}{2}}$ $\quad 1 \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\quad \frac{\cancel{2}\sqrt{2}}{\cancel{2}}$

c) $\theta = \frac{\pi}{3}$

$\frac{\pi}{3} \cdot \frac{180}{\pi}$ 60



$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\qquad \cos\frac{\pi}{3} = \frac{1}{2}$ $\quad \frac{x}{1}$ $\quad \frac{\frac{1}{2}}{1}$ $\qquad \tan\frac{\pi}{3} = \sqrt{3}$ $\quad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\quad \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\quad \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

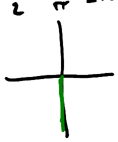
$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$ $\quad \frac{1}{\frac{\sqrt{3}}{2}}$ $\quad 1 \cdot \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\sec\frac{\pi}{3} = 2$ $\quad \frac{1}{x}$ $\quad \frac{1}{\frac{1}{2}}$

$\cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$ $\quad \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\quad \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

d) $\theta = \frac{3\pi}{2}$

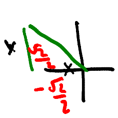
$\frac{3\pi}{2} \cdot \frac{180°}{\pi} = 270$



$\sin\frac{3\pi}{2} = -1$ $\qquad \cos\frac{3\pi}{2} = 0$ $\qquad \tan\frac{3\pi}{2} =$ undefined $\quad \frac{y}{x}$ $\quad \frac{-1}{0}$

$\csc\frac{3\pi}{2} = -1$ $\qquad \sec\frac{3\pi}{2} =$ undefined $\quad \frac{1}{x}$ $\qquad \cot\frac{3\pi}{2} = 0$ $\quad \frac{x}{y}$ $\quad \frac{0}{-1}$

e) $\theta = \frac{3\pi}{4}$



$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{y}{1}$ $\quad \frac{\frac{\sqrt{2}}{2}}{1}$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ $\qquad \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\qquad \tan\frac{3\pi}{4} = -1$

$\csc\frac{3\pi}{4} = \sqrt{2}$ $\quad \frac{1}{\frac{\sqrt{2}}{2}}$ $\quad 1 \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$

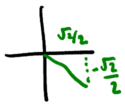
$\sec\frac{3\pi}{4} = -\sqrt{2}$ $\quad 1 \cdot \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\cot\frac{3\pi}{4} = -1$

$\frac{\sqrt{2}}{2}$ $\quad 1\cdot\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$ $\quad 1\cdot\frac{-2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$ [illegible]

f) $\theta = 675°$ $\quad \sin 675° = -\frac{\sqrt{2}}{2}$ $\quad \cos 675° = \frac{\sqrt{2}}{2}$ $\quad \tan 675° = -1$

$675 - 360 = 315$

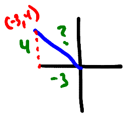


$\csc 675° = -\sqrt{2}$ $\quad \sec 675° = \sqrt{2}$ $\quad \cot 675° = -1$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

Ex. 5 A point on the terminal side of an angle θ is given. Find the exact value of $\sin\theta$ and $\cos\theta$.

a) $(-3, 4)$

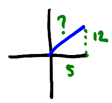


$\sin\theta = \frac{4}{5}$

$(-3)^2 + (4)^2 = c^2$
$25 = c^2$
$c = 5$

$\cos = \frac{-3}{5}$

b) $(5, 12)$



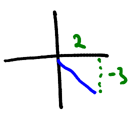
$\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{5}{13}$

$(5)^2 + (12)^2 = c^2$
$169 = c^2$
$c = 13$

c) $(2, -3)$

$\sin\theta = \frac{-3}{\sqrt{13}} \Rightarrow \frac{-3\sqrt{13}}{13}$ $\quad \frac{-3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}}$



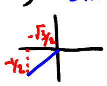
$(2)^2 + (-3)^2 = c^2$
$13 = c^2$
$\sqrt{13} = c$

$\cos\theta = \frac{2}{\sqrt{13}} \Rightarrow \frac{2\sqrt{13}}{13}$

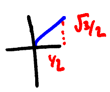
Ex. 6 Determine the exact value.

a) $\sin(-150°) \cdot \sec 60°$

ref: 30

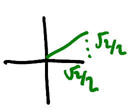


$-\frac{1}{2}\cdot 2 = -1$



$\frac{1}{1/2} = 2$

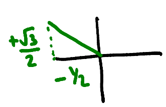
b) $\cos\frac{\pi}{4} - \tan\frac{2\pi}{3}$



$\frac{\sqrt{2}}{2} - (-\sqrt{3})$

$\frac{\sqrt{2}}{2} + \sqrt{3}$

$\frac{\sqrt{2}}{2} + \frac{2\sqrt{3}}{2} \Leftrightarrow \frac{\sqrt{2} + 2\sqrt{3}}{2}$

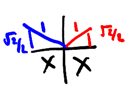


$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$ $\quad \frac{\sqrt{3}}{2}\cdot\frac{-2}{1} = -\sqrt{3}$

Ex. 7 Determine the angle measure in Radians (two angles)

a) $\sin\theta = \frac{\sqrt{2}}{2}$ y-coord.

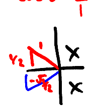
$\sin\theta = \frac{y}{1}$



$\frac{\pi}{4}, \frac{3\pi}{4}$

b) $\cos\theta = -\frac{\sqrt{3}}{2}$ x-coord

$\cos\theta = \frac{x}{1}$



$\frac{5\pi}{6}, \frac{7\pi}{6}$

c) $\tan\theta = -\frac{\sqrt{3}}{3}$

$\tan\theta = \frac{y}{x}$

$\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$\frac{5\pi}{6}, \frac{11\pi}{6}$

d) $\csc\theta = \sqrt{2}$

$\csc\theta = \frac{1}{y}$

$\frac{\pi}{4}, \frac{3\pi}{4}$

Domain of Sine and Cosine functions is the set of all real #'s.

$f(\theta) = \sin\theta$

Range: $-1 \le \sin\theta \le 1$

$-1 \le \cos\theta \le 1$

* Domain is the ∡ measure of θ: $30°, \frac{\pi}{6}, -130°, .78$ rads

Range is the value of the trig. function at a certain ∡ measure: $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1, 0$

A function is Periodic (repeats) if there exists a positive real # c such that $f(\theta + c) = f(\theta)$ for all θ in the domain of f.

Sine and Cosine have a period of 2π or $360°$

$\sin\theta = \sin(\theta \pm 360n)$ ($2\pi n$) $\qquad \cos\theta = \cos(\theta \pm 360n)$ ($2\pi n$)

n: # of periods

Ex. 8 Use the period to change the ∡ measure to an equivalent ∡ measure on the unit circle.

a) $\sin 510°$

$\sin 510° = \sin(510 - 360(1))$

$= \sin 150°$

$\frac{1}{2}$

b) $\sin\frac{13\pi}{4}$

$\sin\frac{13\pi}{4} = \sin\left(\frac{13\pi}{4} - 2\pi\right)$

$= \sin\frac{5\pi}{4}$

$-\frac{\sqrt{2}}{2}$

Ex. 9 Find the exact value of either $\sin\theta$ or $\cos\theta$. (looking for the trig function not given)

a) $\sin\theta = \frac{12}{13}$; $90° < \theta < 180°$

$a^2 + (12)^2 = (13)^2$

$a^2 + 144 = 169$

$a^2 = 25$

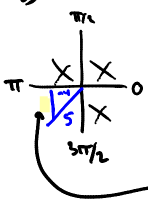
$a = 5$

Actually negative b/c

$\cos\theta = -\frac{5}{13}$

x is in Quad. II.

b) $\cos\theta = \frac{-4}{5}$ $\quad \pi \le \theta \le \frac{3\pi}{2}$



$(-4)^2 + b^2 = (5)^2$

$16 + b^2 = 25$

$b^2 = 9$ $\quad b = 3$

$\sin\theta = -\frac{3}{5}$

Evaluate Trig functions using a TI-83+ or TI-84

$\boxed{\text{MODE}}$ ↓ Radian Degree

Calculator does not have reciprocal functions as keys.

csc → $\boxed{\sin}$ ( ) $\boxed{x^{-1}}$

sec → $\boxed{\cos}$ ( ) $\boxed{x^{-1}}$

cot → $\boxed{\tan}$ ( ) $\boxed{x^{-1}}$

* Make sure you are in the correct MODE! *

Ex. 3 Evaluate using a calculator. (4 decimal places)

Radian

a) $\sin\frac{\pi}{4}$ .7071

b) $\tan\frac{\pi}{3}$ 1.7321

c) $\csc 1.3$ rads   1.0378

d) $\cos(-1.7)$ −.1288

e) $\cot 1$ .6421

f) $\sec 1.8$ −4.4014

Degrees

g) $\cos 173^\circ$ −.9925

h) $\cot -102^\circ$ .2126

j) $\sin 60^\circ$ .8660

k) $\tan 250^\circ$ 2.7475

l) $\sec 120^\circ$ −2

m) $\csc 286^\circ$ −1.0403