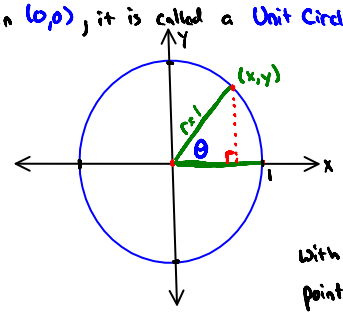
4.2 Unit Circle

Wednesday, March 11, 2015
10:23 AM

When a circle has a radius of 1 and the center is at the origin $(0,0)$, it is called a Unit Circle.



$x^2+y^2=1$ $\quad (x-h)^2+(y-k)^2=r^2$

Center $(0,0)$

$r=1$

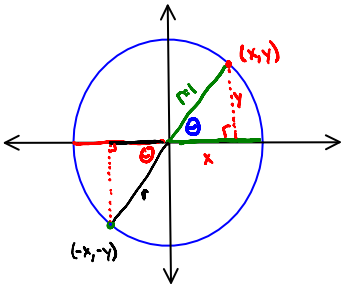
With the unit circle, you can find points on the circle using Rt Δ.

* Always bring a VERTICAL line down or up to the X-axis to form Rt Δ. *

SOH	CAH	TOA	
$\sin=\frac{O}{H}$	$\cos=\frac{A}{H}$	$\tan=\frac{O}{A}$	} Right Triangles
$\csc=\frac{H}{O}$	$\sec=\frac{H}{A}$	$\cot=\frac{A}{O}$	

Six Trig Functions and the Unit Circle; where $r=1$

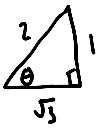
Function	Unit Circle Relation	
Sine	$\sin=\frac{y}{1}$	$\frac{y}{r}$
Cosine	$\cos=\frac{x}{1}$	$\frac{x}{r}$
Tangent	$\tan=\frac{y}{x}$	$\frac{\sin\theta}{\cos\theta}$
Cosecant	$\csc=\frac{1}{y}$	$\frac{r}{y}$ $\quad \frac{1}{\sin\theta}$
Secant	$\sec=\frac{1}{x}$	$\frac{r}{x}$ $\quad \frac{1}{\cos\theta}$
Cotangent	$\cot=\frac{x}{y}$	$\frac{\cos\theta}{\sin\theta}$

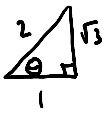


Notice that the y-coord is $\sin\theta$ and the x-coord is $\cos\theta$; only on the unit circle. $(x,y) \rightarrow (\cos\theta, \sin\theta)$

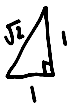
Cos θ ; only on the unit circle. $(x, y) \rightarrow (\cos\theta, \sin\theta)$

Review Special Right Triangles

$\theta = 30°$ or $\frac{\pi}{6}$ 

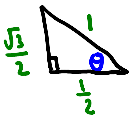
$\theta = 60°$ or $\frac{\pi}{3}$ 

For 30°-60°, hypotenuse is twice the short side. The middle leg is $\sqrt{3}$ · short side

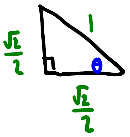
$\theta = 45°$ or $\frac{\pi}{4}$ Isosceles 

For 45°, hypotenuse is $\sqrt{2}$ · short side.

Special Right Δs on the unit Circle (Reference triangles)

$\theta = 60°$ or $\frac{\pi}{3}$ 

$\theta = 30°$ or $\frac{\pi}{6}$ 

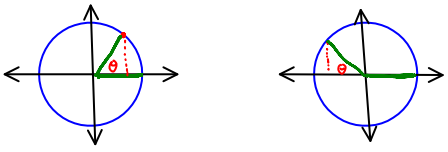
$\theta = 45°$ or $\frac{\pi}{4}$ 

Study Filled out Unit Circle

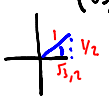
When x=0 tangent and secant functions are undefined.
Happens at 90° or $\frac{\pi}{2}$ and 270° or $\frac{3\pi}{2}$

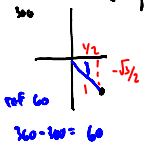
When y=0 cotangent and cosecant functions are undefined.
Happens at 0° or 0 rad, 180° or π, 360° or 2π

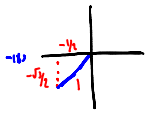
Reference Angle is an acute angle formed by the terminal side and x-axis

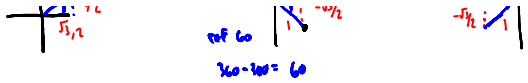


Ex. 1 What ordered pair on the Unit ⊙ corresponds to the following angle.

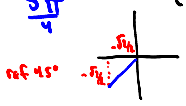
a) $\frac{\pi}{6}$ 30° $(\sqrt{3}/2, 1/2)$ 

b) $\frac{5\pi}{3}$ 300 $(1/2, -\sqrt{3}/2)$ 

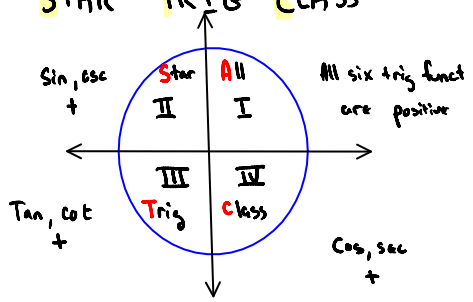
c) $-\frac{2\pi}{3}$ -120 $(-1/2, -\sqrt{3}/2)$ 



d) $\frac{5\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

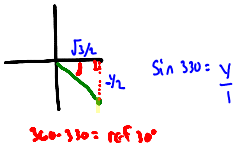


ALL STAR TRIG CLASS

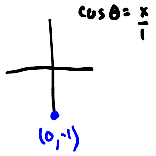


Ex. 2 Give exact values

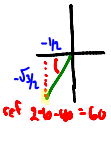
a) $\sin 330° = -\frac{1}{2}$



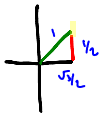
b) $\cos 270° = 0$



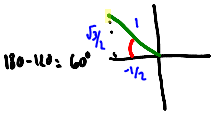
c) $\sin 240 = \frac{-\sqrt{3}}{2}$



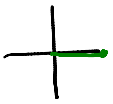
d) $\cos 30° = \frac{\sqrt{3}}{2}$



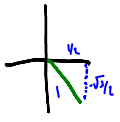
e) $\cos 120° = -\frac{1}{2}$



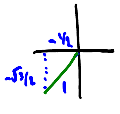
f) $\sin 0° = 0$



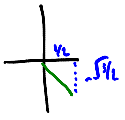
g) $\cos 300° = \frac{1}{2}$



h) $\sin -120° = -\frac{\sqrt{3}}{2}$



i) $\cos -60° = \frac{1}{2}$

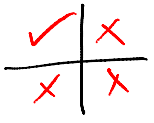


Ex. 3 What quadrant does θ lie?

a) $\sin\theta < 0$, $\cos\theta > 0$

IV

b) $\tan\theta < 0$, $\cos\theta < 0$



II

c) $\csc\theta > 0$, $\sec\theta > 0$

I

d) $\sec\theta < 0$, $\sin\theta < 0$

III

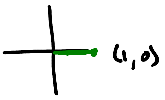
c) $\cot\theta > 0$, $\sin\theta < 0$

III

Ex. 4 Evaluate for each of the six trig. functions

a) $\theta = 0°$

$\sin\theta = 0$ $\quad \cos\theta = 1$ $\quad \tan\theta = 0$ $\quad \frac{0}{1} = 0$



$\csc\theta$ = undefined $\quad \sec\theta = 1$ $\quad \cot\theta$ = undefined

Reciprocals of the first 3!

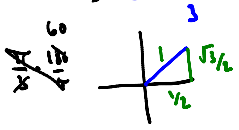
b) $\theta = 45°$

$\sin 45° = \frac{\sqrt{2}}{2}$ $\quad \cos 45° = \frac{\sqrt{2}}{2}$ $\quad \tan 45° = 1$ $\quad \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$\csc 45° = \sqrt{2}$ $\quad \sec 45° = \sqrt{2}$ $\quad \cot 45° = 1$

$\frac{1}{\frac{\sqrt{2}}{2}}$ $\quad 1 \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ $\quad \frac{2\sqrt{2}}{2}$

c) $\theta = \frac{\pi}{3}$



$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \cos\frac{\pi}{3} = \frac{1}{2}$ $\quad \tan\frac{\pi}{3} = \sqrt{3}$

$\frac{\frac{1}{2}}{1}$ $\quad \frac{x}{1}$ $\quad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\quad \frac{\sqrt{3}}{2} \cdot \frac{2}{1}$

$\csc\frac{\pi}{3} = \frac{2\sqrt{3}}{3}$ $\quad \sec\frac{\pi}{3} = 2$ $\quad \cot\frac{\pi}{3} = \frac{\sqrt{3}}{3}$

$\frac{1}{\frac{\sqrt{3}}{2}}$ $\quad 1 \cdot \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ $\quad \frac{1}{x}$ $\quad \frac{1}{\frac{1}{2}}$ $\quad \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\quad \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\quad \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

d) $\theta = \frac{3\pi}{2}$

$\frac{3\pi}{2} \cdot \frac{180°}{\pi} = 270$

$\sin\frac{3\pi}{2} = -1$ $\quad \cos\frac{3\pi}{2} = 0$ $\quad \tan\frac{3\pi}{2}$ = undefined $\quad \frac{-1}{0}$

$\frac{y}{x}$

$\csc\frac{3\pi}{2} = -1$ $\quad \sec\frac{3\pi}{2}$ = undefined $\quad \cot\frac{3\pi}{2} = 0$

$\frac{1}{x}$ $\quad \frac{x}{y}$ $\quad \frac{0}{-1}$

e) $\theta = \frac{3\pi}{4}$

$\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\quad \tan\frac{3\pi}{4} = -1$

$\frac{y}{1}$ $\quad \frac{\frac{\sqrt{2}}{2}}{1}$

$\csc\frac{3\pi}{4} = \sqrt{2}$ $\quad \sec\frac{3\pi}{4} = -\sqrt{2}$ $\quad \cot\frac{3\pi}{4} = -1$

$\frac{1}{}$

$-\frac{\sqrt{2}}{2}$ |

$\csc\frac{3\pi}{4} = \sqrt{2}$ $\quad \sec\frac{3\pi}{4} = -\sqrt{2}$ $\quad \cot\frac{3\pi}{4} = -1$

$\frac{1}{\frac{\sqrt{2}}{2}} \quad 1 \cdot \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2}$ $\quad 1 \cdot -\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

f) $\theta = 675°$ $\quad \sin 675° = -\frac{\sqrt{2}}{2}$ $\quad \cos 675° = \frac{\sqrt{2}}{2}$ $\quad \tan 675° = -1$

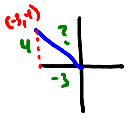
$675 - 360 = 315$

$\csc 675° = -\sqrt{2}$ $\quad \sec 675° = \sqrt{2}$ $\quad \cot 675° = -1$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

Ex. 5 A point on the terminal side of an angle θ is given. Find the exact value of $\sin\theta$ and $\cos\theta$.

a) $(-3, 4)$



$(-3)^2 + (4)^2 = c^2$

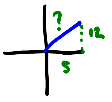
$25 = c^2$

$c = 5$

$\sin\theta = \frac{4}{5}$

$\cos = \frac{-3}{5}$

b) $(5, 12)$



$(5)^2 + (12)^2 = c^2$

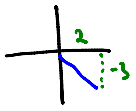
$169 = c^2$

$c = 13$

$\sin\theta = \frac{12}{13}$

$\cos\theta = \frac{5}{13}$

c) $(2, -3)$ $\quad \sin\theta = \frac{-3}{\sqrt{13}} \Rightarrow \frac{-3\sqrt{13}}{13}$ $\quad \frac{-3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$



$(2)^2 + (-3)^2 = c^2$

$13 = c^2$

$\sqrt{13} = c$

$\cos\theta = \frac{2}{\sqrt{13}} \Rightarrow \frac{2\sqrt{13}}{13}$

Ex. 6 Determine the exact value.

a) $\sin(-150°) \cdot \sec 60°$

ref: 30

$-\frac{1}{2} \cdot 2 = -1$

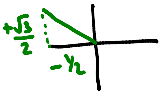
$\frac{1}{1/2} = 2$

b) $\cos\frac{\pi}{4} - \tan\frac{2\pi}{3}$

$\frac{\sqrt{2}}{2} - (-\sqrt{3})$

$\frac{\sqrt{2}}{2} + \sqrt{3}$

$\frac{\sqrt{2}}{2} + \frac{2\sqrt{3}}{2} \Leftrightarrow \frac{\sqrt{2} + 2\sqrt{3}}{2}$



$\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} \quad \frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = -\sqrt{3}$