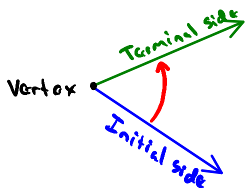
4.1 Radian and Degree Measure

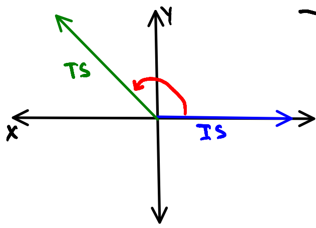
Monday, March 09, 2015
9:41 AM

Trigonometry is the Greek word that means "measurement of triangles."

An <u>**Angle**</u> is determined by rotating a RAY about its end points; it has 2 sides:

- **Initial Side** is the starting point of the ray.
- **Terminal Side** is the position after rotation.
- Vertex is the end point of the ray.





An angle that is in **Standard Position** has its vertex at the origin and the initial side coincides with the positive x-axis.

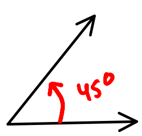
Angles are labeled with <u>Greek letters</u> and UPPERCASE LETTERS:

α	β	γ	θ
Alpha	Beta	Gamma	Theta

Angles are identified by showing the **direction** and the amount of **Rotation** from the initial side.

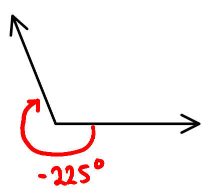
<u>**Positive Angles**</u>

Are formed by a **counter clockwise** rotation.



<u>**Negative Angles**</u>

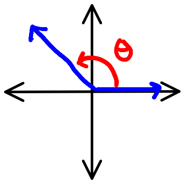
Are formed by a **clockwise** rotation.



When $\angle\theta$ is in Standard Position, the terminal side will be in any of the 4 quadrants of a coordinate plane.

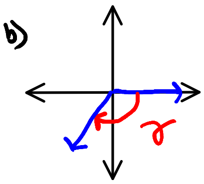
Ex. 1 Identify the quadrant of the angle and whether its measure is positive or negative.

a)



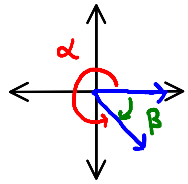
Quad. II, Positive ∠

b)



Quad III, negative ∠

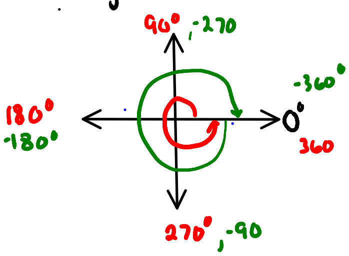
c)



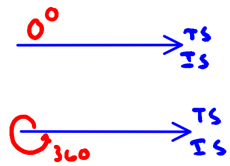
α Quad. IV, positive ∠
β Quad. IV, negative ∠

Angles α and β have the same I.S. and T.S., these angles are called Coterminal Angles.

Angles are measured in Degrees and Radians.



If the I.S. and T.S. are the same ray, then the degree measure is either:

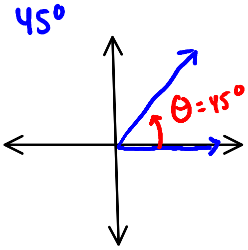


One revolution of a ray is 360°

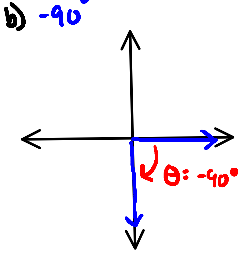
- 1° is 1/360 revolution
- 90° is 90/360 is 1/4 revolution
- 180° is 180/360 is 1/2 revolution
- 270° is 270/360 is 3/4 revolution
- 360° is 360/360 is 1 revolution

Ex.2 Draw each angle

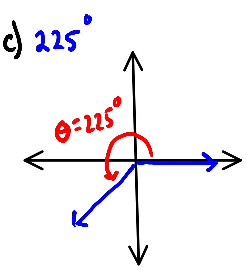
a) 45°



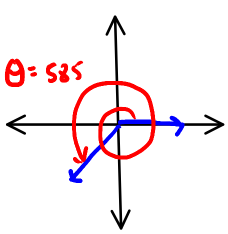
b) -90°



c) 225°

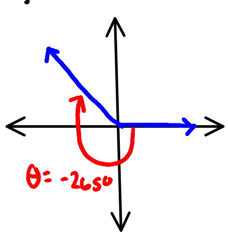


d) 585°

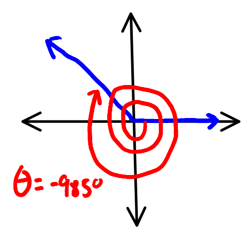


585
-360
225°

e) -265°

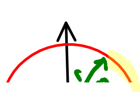


f) -985°

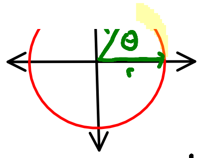


-985
+360
-625
+360
-265°

A second way to measure angles is in Radians. It is a unit of angular measurement.



Measure of a central angle θ that



intercepts an arc equal in length to the radius of the circle.

When arc length equals radius, θ equals 1 radian.

$C = 2\pi r$ (circumference of a circle) when $r = 1$, then the circumference is 2π. So arc length of

arc length → $S = 2\pi$.

2π radians corresponds to 360°
π radians corresponds to 180°

Radian is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle.

1 radian corresponds to 57.296° to an arc length of 1 corresponds to $\frac{180}{\pi}$

Use for conversion: $\frac{180}{\pi} = 1$ rad or $\frac{\pi}{180} = 1°$

Ex.3 Convert from degrees to Radians

a) 60°
$60 \cdot \frac{\pi}{180}$
$\frac{60\pi}{180} \rightarrow \frac{\pi}{3}$ rads

b) 150°
$150 \cdot \frac{\pi}{180}$
$\frac{5\pi}{6}$ rads

c) −45°
$-45\left(\frac{\pi}{180}\right)$
$-\frac{\pi}{4}$ rads

d) 120°
$120\left(\frac{\pi}{180}\right)$
$\frac{2\pi}{3}$ rads

Ex.4 Convert from Radians to degrees

a) $\frac{\pi}{5}$
$\frac{\pi}{5} \cdot \frac{180}{\pi}$
$\frac{180}{5} \rightarrow 36°$

b) $\frac{7\pi}{4}$
$\frac{7\pi}{4} \cdot \frac{180}{\pi}$
315°

c) $-\frac{\pi}{3}$
$-\frac{\pi}{3} \cdot \frac{180}{\pi}$
−60°

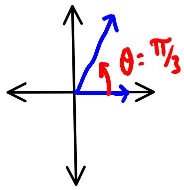
d) $-\frac{7\pi}{3}$
$-\frac{7\pi}{3} \cdot \frac{180}{\pi}$
−420°

e) .682 radians
$.682 \cdot \frac{180}{\pi}$
$\frac{122.76}{\pi}$
39.08°

Ex.5 Draw the angles

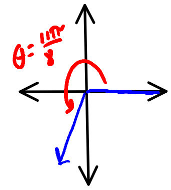
a) π b) 11π c) 5π

a) $\frac{\pi}{3}$



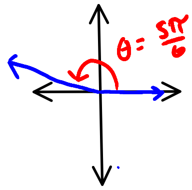
$\frac{\pi}{3} \cdot \frac{180}{\pi} = 60°$

b) $\frac{11\pi}{8}$



$\frac{11\pi}{8} \cdot \frac{180}{\pi} = 247.5°$

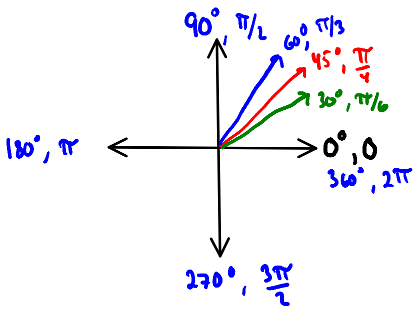
c) $\frac{5\pi}{6}$



$\frac{5\pi}{6} \cdot \frac{180}{\pi} = 150°$

Degrees	0°	30°	45°	60°	90°	180°	270°	360°
Radians	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π

} memorize



Converting Degrees Minutes Seconds

One minute (1′) is defined as $\frac{1}{60}$ degrees

One second (1″) is defined as $\frac{1}{60}$ minutes or $\frac{1}{3600}$ degrees

Ex. 6

a) Convert 35° 11′12″ to a decimal in degree.

$35° + 11(1/60) + 12(1/3600) = 35.1867°$

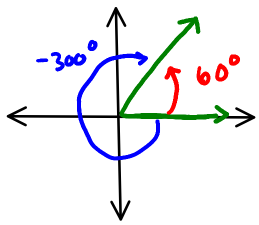
b) Convert 73.479° to DMS

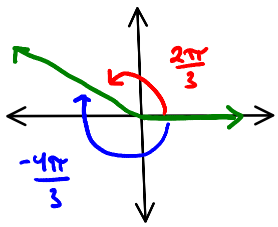
Degree	Minutes first	Seconds
73°	.479 (60)	.74 (60)
	28.74	44.4

73° 28′ 44.4″

Coterminal Angles

Are 2 angles in Standard Position that have the same I.S. and T.S.





You can find an angle that is coterminal to a given angle by adding or subtracting 360° or 2π (1 revolution).

* There are infinitely many coterminal angles *

ex: 30°

$30 + 360 = 390°$

$30 + 360(-1) = -330°$

$\frac{\pi}{6}$

$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$

$\frac{\pi}{6} + 2\pi(-1) = -\frac{11\pi}{6}$

Ex. 7 Find two positive and 2 negative angles coterminal with the given angle of measure:

a) 60°

$60 + 360(1) = 420°$
$60 + 360(2) = 780°$ } Positive

$60 + 360(-1) = -300°$
$60 + 360(-2) = -660$ } Negative

b) $\frac{13\pi}{4}$

$\frac{13\pi}{4} + 2\pi(1) = \frac{21\pi}{4}$
$\frac{13\pi}{4} + 2\pi(2) = \frac{29\pi}{4}$ } Positive

$\frac{13\pi}{4} + 2\pi(-1) = \frac{5\pi}{4}$ * Positive not negative

$\frac{13\pi}{4} + 2\pi(-2) = -\frac{3\pi}{4}$

$\frac{13\pi}{4} + 2\pi(-3) = -\frac{11\pi}{4}$

Complementary Angles

Are two angles that add up to 90° or $\frac{\pi}{2}$.

Supplementary Angles

Are two angles that add up to 180° or π.

Ex. 8 Find the complement and supplement to the given angle.

Ex. 8 Find the complement and supplement to the given angle.

a) $65°$

Comp.
$90 - 65 = 25°$

Supp.
$180 - 65 = 115°$

b) $105°$

Comp.
$90 - 105 = -15°$

None

Supp.
$180 - 105 = 75°$

c) $\frac{\pi}{7}$

comp.
$\frac{\pi}{2} - \frac{\pi}{7} = \frac{5\pi}{14}$

Supp.
$\pi - \frac{\pi}{7} = \frac{6\pi}{7}$

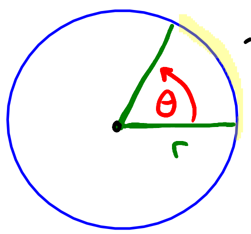
d) $\frac{2\pi}{3}$

comp.
$\frac{\pi}{2} - \frac{2\pi}{3} = -\frac{\pi}{6}$

None

Supp.
$\pi - \frac{2\pi}{3} = \frac{\pi}{3}$

Arc Length (s)



$S = r\theta$

* θ has to be in radians *

FYI
$\text{deg} \cdot \frac{\pi}{180} = \text{radians}$

Ex. 1

a) $S = r\theta$
$r = 6\text{ m}$ $\theta = .35\text{ rad}$

$S = 6(.35)$
$S = 2.1\text{ m}$

b) $S = r\theta$
$r = ?$ 20 in diameter $\frac{20}{2} = 10\text{ in}$
$\theta = 250° \rightarrow 250 \cdot \frac{\pi}{180} = \frac{25\pi}{18}$

$S = 10\left(\frac{25\pi}{18}\right)$

$S = 43.63\text{ in}$

c) $S = r\theta$
$S = 5\text{m}$ $r = 1.5\text{m}$ $\theta = ?$
$5 = 1.5\theta$
$\theta = 3.33\text{ rads}$
$10/3\text{ rads}$

degrees
$10/3\left(\frac{180}{\pi}\right)$ 3.33

$190.99°$ 190.79

Linear Speed

Linear speed $v = \frac{S}{t}$ (S: arc length, t: time)

Angular Speed

Angular speed $\omega = \frac{\theta}{t}$ (θ: central ∡ (radians), t: time)

Speed $\frac{}{t}$ time   Angular speed $\frac{}{t}$ time

Ex. 2 Applications

a) $v = \frac{s}{t}$

$s = r\theta$

$r = 12.5 \quad \theta = 2\pi$

$s = 12.5(2\pi)$

$s = 25\pi$

$v = \frac{25\pi \text{ cm}}{60 \text{ sec}}$

$v = 1.31$ cm/sec

b) $v = \frac{s}{t}$

$s = r\theta$

$s = 12(7.2\pi)$

$s = 168\pi$ cm

$v = \frac{168\pi \text{ cm}}{1 \text{ sec}}$

$v = 527.79$ cm/sec

c) $v = \frac{s}{t}$

$s = r\theta$

$r = 18\text{in}$ convert to ft $\rightarrow 18\text{in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1.5$ ft

convert $36 \frac{\text{rev}}{\text{min}}$ to sec $\rightarrow \frac{36 \text{ rev}}{1 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = .6$ rev/sec

$\theta = .6(2\pi) = 1.2\pi$

$s = 1.5(1.2\pi) = 1.8\pi$ ft

$v = \frac{1.8\pi}{1 \text{ sec}}$

$v = 5.65$ ft/sec

d) $\omega = \frac{\theta}{t}$

$\theta = 1.2(2\pi)$

$= 2.4\pi$

$\omega = \frac{2.4\pi \text{ in}}{1 \text{ sec}}$

$\omega = 2.4\pi$ rad/sec

e) $\omega = \frac{\theta}{t}$

$\theta = 6.8(2\pi)$

$= 13.6\pi$

$\omega = 13.6\pi$

$$\overline{8\text{ sec}}$$

$$\omega = 1.7\pi \frac{\text{rad}}{\text{sec}}$$