3.5 Exponential and Logarithmic Models

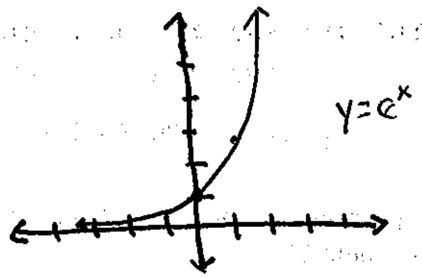
The **five most common** types of mathematical models involving exponential functions and logarithmic functions are:

1. **Exponential Growth**

$y = ae^{bx}$ $\quad b>0$

$y = ab^x$

$b > 1$

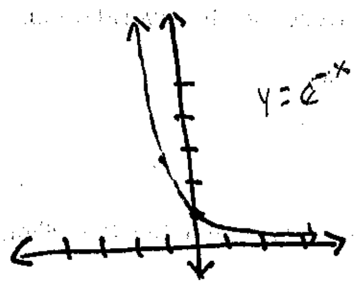


2. **Exponential Decay**

$y = ae^{-bx}$ $\quad b>0$

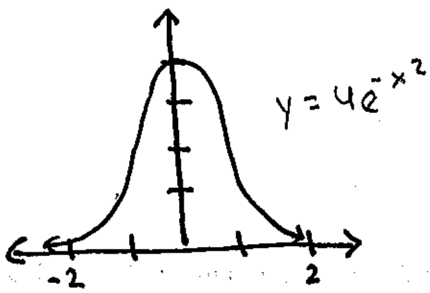
$y = ab^x$

$0 < b < 1$



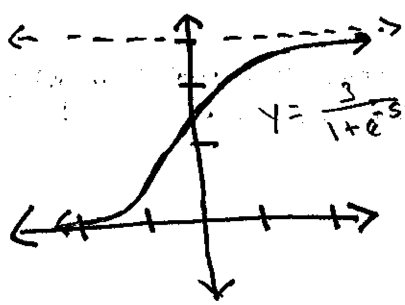
3. **Gaussian Model**

$y = ae^{-(x-b)^2/c}$



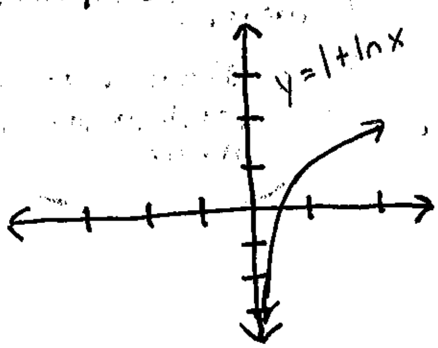
4. **Logistic Growth**

$y = \dfrac{a}{1+be^{-rx}}$

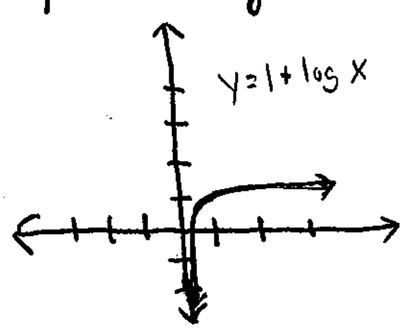


5. **Logarithmic Models**

$y = a + b\ln x$



$y = a + b\log x$



3.5 Exponential and Logarithmic Models

Ex. 1

The estimates for the world population (in millions) from 1992 through 2000 are shown in the table below:

	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9	t=10
Year	1992	1993	1994	1995	1996	1997	1998	1999	2000
Population	5445	5527	5607	5688	5767	5847	5926	6005	6083

$2 \le t \le 10$

An exponential growth model that approximates this data is $P(t) = 5303e^{0.013938t}$, where *P(t)* is the population (in millions) and **t=2 represents 1992.**

Chart starts @ t=2 so t=0 is 1990.

a) According to this model, what year corresponds to a world population of 6.5 billion?

P=6.5 billion

P= 6.5 (1,000)

P = 6500 million

$\frac{6500}{5303} = \frac{5303}{5303} e^{.013938t}$

$1.2257 = e^{.013938t}$

$\ln 1.2257 = .013938t$

$t = \frac{\ln 1.2257}{.013938}$

$t \approx 14.60$ yrs

t=2 1992

$14.60 - 2.00 = 12.60$

$1992 + 12.60 = 2004.6$

t=0 1990

$1990 + 14.6 = 2004.6$

In 2004, mid-year

b) In what year will the world population be approximately 8.5 billion?

P = 8.5 billion

P= 8.5 (1000)

P = 8500

$\frac{8500}{5303} = \frac{5303}{5303} e^{.013938t}$

$1.6029 = e^{.013938t}$

$\frac{\ln 1.6029}{0.013938} = \frac{0.013938t}{0.013938}$

$t \approx 33.85$ yrs

t=0 1990

$1990 + 33.85 = 2023.85$

In late 2023!

Ex. 2

The number of deer in a wildlife preserve was estimated to be continuously increasing at the yearly rate of 12%. ← r

$A = Pe^{rt}$

a) For each 1000 deer now in the preserve, estimate the number that will be present after 6 months.

$A = Pe^{rt}$

P=1000

r=12% → .12

t= 6 months → .5

$A = 1000e^{.12(.5)}$

$A \approx 1061.8365$

$A \approx 1061$ deer

fyi

deer, bacteria, people } Round down to whole #

time, grams, money } Round to 2 decimal places

* IF not a word problem round to four decimal places

b) How many years will it take the deer population to triple?

A = 3000

P = 1000

r = 12%

t = ?

$3000 = 1000e^{.12t}$

$3 = e^{.12t}$

$\ln 3 = .12t$

$t = \frac{\ln 3}{.12}$

$t \approx 9.16$ yrs

3.5 Exponential and Logarithmic Models

Ex. 3

In a research experiment, a population of fruit flies is increasing in accordance with the law of exponential growth. After five days there are 500 flies and after eight days there are 700 flies. $y = ae^{bt}$

5 (5, 500) 8 (8, 700)

a) How many flies will there be after 10 days?

Find "a" first

$500 = ae^{5b}$ $700 = ae^{8b}$

$a = \frac{500}{e^{5b}}$ ratio $a = \frac{700}{e^{8b}}$

find b

$\frac{700}{e^{8b}} = \frac{500}{e^{5b}}$

$700 = \frac{500(e^{8b})}{e^{5b}} \rightarrow 700 = 500e^{3b}$

$\frac{700}{500} = e^{3b} \rightarrow \ln\frac{7}{5} = 3b$ $b \approx .1122$

$y = \left(\frac{500}{e^{5(.1122)}}\right)e^{.1122(10)}$

$y \approx 285.38e^{.1122(10)}$

$y \approx 876$ flies

Ex. 4

Carbon 14 decomposes radioactively and it has a half-life of 5750 years. $A = Pe^{rt}$ or $y = ae^{xt}$

a) An archaeologist determines that a certain bone retains 60% of its original amount of Carbon 14. How old is the bone?

$= t?$ $50\% = .5$ half life

First find rate of decay

$.5 = 1e^{r(5750)}$

$.5 = e^{r(5750)}$

$\ln .5 = 5750r$

$\frac{\ln .5}{5750} = r \rightarrow r \approx -1.2055 \times 10^{-4}$

Sci Not # in calculator

$A = Pe^{rt}$ $P = 100\% \rightarrow 1$

$A = 60\% \rightarrow .60$

$.6 = 1e^{(-1.2055\times10^{-4})(t)}$

$\ln .6 = (-1.2055 \times 10^{-4})t$

$\frac{\ln .6}{-1.2055\times10^{-4}} = t$ $t \approx 4237.96$ yrs

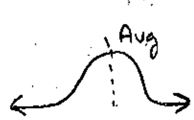
Gaussian Models:

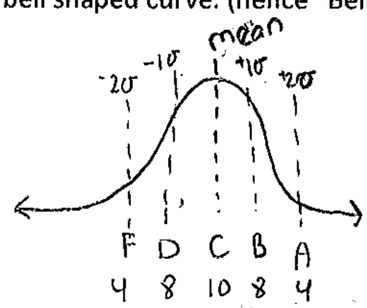
Used commonly in probability and statistics to represent populations that is normally distributed. For standard normal distributions, the model takes the form:

$$y = \frac{1}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2}$$

σ = standard deviation

The graph is a bell shaped curve. (hence "Bell Curve")





3.5 Exponential and Logarithmic Models

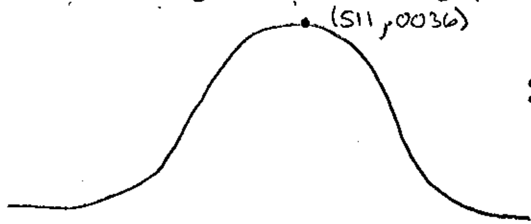
Ex. 5

In 2002, the SAT Math scores roughly followed the normal distribution:

$y = 0.0036e^{-(x-511)^2/25088}$, $200 \le x \le 800$, where x = SAT score Graph

a) Estimate the average SAT score from a graph.

$Y_1 = .0036e^{\wedge}((-(x-511)^2)/25088)$



See page 262 Graph

1) must set window parameters,
2) then zoom 0 (Fit)
3) 2nd Trace max

Logistic Model:

This model type is that of growth that has an initially rapid growth, followed by a declining rate of growth (also called **sigmoidal curve**).

$$y = \frac{a}{1+be^{-rx}}$$

Ex. 6

On a college campus of 5000 students, one student returns from vacation with a contagious flu virus. The spread of the virus is modeled by $y = \frac{5000}{1+4999e^{-0.8t}}$, where *y* is the total number infected after *t* days. The college will cancel classes when 40% or more of the students are ill.

a) How many students are infected after 5 days?

$$y = \frac{5000}{1+4999e^{-.8(5)}}$$

$y \approx 54$ students

b) After how many days will the college cancel classes?

40% of 5000

.40(5000)

2000

$(1+4999e^{-.8t})2000 = \frac{5000}{1+4999e^{-.8t}}(1+4999e^{-.8t})$

$\frac{(1+4999e^{-.8t})2000}{2000} = \frac{5000}{2000} \rightarrow 1+4999e^{-.8t} = 2.5$

$4999e^{-.8t} = 1.5$

$e^{-.8t} = \frac{1.5}{4999}$

$-.8t = \ln\frac{1.5}{4999}$

$t \approx 10.14$ days

Ex. 7

On the Richter scale, the magnitude *R* of an earthquake of intensity *I* is $R = \log\frac{I}{I_0}$, where I_0 is the minimum intensity used for comparison. Find the intensities per unit of area for the following earthquakes. (Intensity is a measure of the wave energy of the earthquake.)

$I_0 = 1$

a) Tokyo and Yokohama, Japan in 1923; R = 8.3

$R = \log\frac{I}{I_0}$ $\quad 8.3 = \log\frac{I}{1}$ $\quad 8.3 = \log I$ $\quad \log I = 8.3$ $\quad 10^{8.3} = I$ $\quad I \approx 199{,}526{,}231.5$

b) Izmit, Turkey, in 1999; R ± 7.4

$R = \log\frac{I}{I_0}$ $\quad 7.4 = \log I$ $\quad 10^{7.4} = I$ $\quad I \approx 25{,}118{,}864.32$

$$\frac{199\,526\,231.5}{25{,}118{,}864.32}$$

≈ 8 times greater