3.4 Solving Exponential and Logarithmic Equations

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11:51 AM

Two basic strategies for solving exponential / logarithmic equations:

1) One-to-One Property $a^x = a^y$ then $x = y$
2) Inverse Property $\log_a a^x \to x$ $a^{\log_a x} \to x$

Ex. 1 Solve

a) $3^x = 81$
$3^x = 3^4$
$x = 4$
$\{4\}$

b) $(1/5)^x = 125$
$(5^{-1})^x = 5^3$
$-x = 3$
$x = -3$
$\{-3\}$

c) $\ln x - \ln 7 = 0$
$\ln x = \ln 7$
$x = 7$
$\{7\}$

d) $4^x = 35$
$\log 4^x = \log 35$
$x \frac{\log 4}{\log 4} = \frac{\log 35}{\log 4}$
$x = \frac{\log 35}{\log 4}$
$x \approx 2.5646$
$\{2.5646\}$

If you have a variable exponent but cannot make like bases, take the log or ln of both sides.

$\log_4 35 = x$
$\frac{\log 35}{\log 4} = x$

e) $e^x = 15$
$\ln e^x = \ln 15$
$x = \ln 15$
$x \approx 2.7081$
$\{2.7081\}$

f) $\frac{2(3^x)}{2} = \frac{58}{2}$
$3^x = 29$
$\log 3^x = \log 29$
$x \log 3 = \log 29$
$x = \frac{\log 29}{\log 3}$
$x \approx 3.065$
$\{3.065\}$

g) $3(4^{2m-5}) - 5 = 17$
$\frac{3(4^{2m-5})}{3} = \frac{22}{3}$
$4^{2m-5} = \frac{22}{3}$
$\log 4^{2m-5} = \log (22/3)$
$2m - 5 = \frac{\log(22/3)}{\log 4}$
$2m = \frac{\log(22/3)}{\log 4} + 5$
$2m = 6.4372$
$m = 3.2186$
$\{3.2186\}$

$\{3.2186\}$

Factor, ZPP, take log or ln

h) $2e^{2x}-5e^{x}-12=0$

$-5e^x$	-24
$3e^x \cdot -8e^x$	$3 \cdot -8$

$(2e^{2x}+3e^{x})(-8e^{x}-12)=0$
$e^{x}(2e^{x}+3)-4(2e^{x}+3)=0$
$(e^{x}-4)(2e^{x}+3)=0$

$e^{x}-4=0$
$e^{x}=4$
$\ln e^{x}=\ln 4$
$x=1.3863$

$2e^{x}+3=0$
$2e^{x}=-3$
$e^{x}=-3/2$
$\ln e^{x}=\ln \cancel{-3/2}$ $\quad x>0$

$\{1.3863\}$

j) $e^{2x}-3e^{x}+2=0$

*Must ✓ your solutions *

$(e^{2x}-2e^{x})(-e^{x}+2)=0$
$e^{x}(e^{x}-2)-1(e^{x}-2)=0$
$(e^{x}-1)(e^{x}-2)=0$

$e^{x}-1=0$
$e^{x}=1$
$\ln e^{x}=\ln 1$
$x=0$

$e^{x}-2=0$
$e^{x}=2$
$\ln e^{x}=\ln 2$
$x=\ln 2$
$x=.6931$

$\{0, .6931\}$

Ex. 2 Solve

a) $\log_5 x=4$
$5^4=x$
$625=x$
$\{625\}$

b) $\ln x=4$
$\log_e x=4$
$e^4=x$
$x=54.5982$
$\{54.5982\}$

c) $\log_6 4=x$
$\frac{\log 4}{\log 6}=x$
$x\approx .7737$
$\{.7737\}$

"Use change-of-base formula".

d) $\ln(2x-5)=\ln(7x+27)$

$2x-5=7x+27$

domain $x>0$!

$-5x=32$

$x=-6.4$

$\emptyset$

e) $\log_4(6x-3)=\log_4(x+17)$

$6x-3=x+17$

$5x=20$

$x=4$

$\{4\}$

f) $\ln(x-2)+\ln(2x-3)=2\ln(x)$

$\ln(x-2)(2x-3)=\ln x^2$

$2x^2-7x+6=x^2$

$x^2-7x+6=0$

$(x-6)(x-1)=0$

$x-6=0 \quad x-1=0$

$x=6 \quad x=1$

*Must ✓ your solution(s).

extraneous

$\{6\}$

g) $\log_{14} x = 2/3$

$14^{2/3}=x$

$x=5.8088$

$\{5.8088\}$

h) $\log_8(x^2-14)=\log_8(5x)$

$x^2-14=5x$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$x-7=0 \quad x+2=0$

$x=7 \quad \cancel{x=-2}$

$\{7\}$

j) $\frac{2\log_5 3x}{2}=\frac{4}{2}$

$\log_5 3x=2$

$5^2=3x$

$25=3x$

$x=25/3$

$\{8.3333\}$

k) $6+2\ln x=9$

$2\ln x=3$

$\ln x=3/2$

$\log_e x=3/2$

$\ln x \rightarrow \log_e x$

$e^{3/2}=x$

$x\approx 4.4817$

$\{4.4817\}$

Can solve logarithmic equations using a graphing utility.

Put left side in $y_1=$ and right side in $y_2=$ then use $\boxed{2nd}$ $\boxed{trace}$ 5: intersect.

Ex. 3 Solve using graphing utility

a) $\ln x = x^2 - 4x$

$y_1 = \ln x$

$y_2 = x^2 - 4x$

$\{.3141, 4.3383\}$

b) $2\log_5 3x = 4$

$y_1 = 2\left(\frac{\log 3x}{\log 5}\right)$

must use change-of-base formula.

$y_2 = 4$

$\{25/3\}$

Ex. 4

a) You have deposited \$750 into an account that pays 5.75% interest compounded continuously. How long will it take for my investment to double?

$A = Pe^{rt}$

$\frac{1500}{750} = \frac{750}{750}e^{.0575t}$

$2 = e^{.0575t}$

$\ln 2 = \ln e^{.0575t}$

$\frac{\ln 2}{.0575} = \frac{.0575t}{.0575}$

$t \approx 12.0547$

Approx. 12 years

b) From 1970 to 1997, the consumer Price Index (CPI) value y for a fixed amount of sugar for year t can be modeled by the eqn:

$y = -171.8 + 87.1\ln t$

where y=10 represents 1970. During which year did the price of sugar reach 4.5 times its 1970 price of \$28.80 on the CPI?

$28.80 \times 4.5 = 129.6$

$129.6 = -171.8 + 87.1\ln t$

$\frac{301.4}{87.1} = \frac{87.1\ln t}{87.1}$

$\frac{301.4}{87.1} = \ln t \rightarrow \frac{301.4}{87.1} = \log_e t$

$e^{(301.4/87.1)} = t$

31.8
− 10

$t = 31.8$ yrs

21.8 yrs + 1970 = 1991.8

$.8 \times 12 = 9.6$

$.6 \times 30$

1991, september 18th