

# 3.4 Solving Exponential and Logarithmic Equations

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11:51 AM

Two basic strategies for solving exponential / logarithmic equations:

1) One-to-One Property

$$a^x = a^y \text{ then } x=y$$

2) Inverse Property

$$\log_a a^x \rightarrow x$$

$$a^{\log_a x} \rightarrow x$$

## Ex.1 Solve

a)  $3^x = 81$

$$3^x = 3^4$$

$$x = 4$$

$$\{4\}$$

b)  $(1/5)^x = 125$

$$(5^{-1})^x = 5^3$$

$$-x = 3$$

$$x = -3$$

$$\{-3\}$$

c)  $\ln x - \ln 7 = 0$

$$\ln x = \ln 7$$

$$x = 7$$

$$\{7\}$$

d)  $4^x = 35$

$$\log 4^x = \log 35$$

$$x \frac{\log 4}{\log 4} = \frac{\log 35}{\log 4}$$

$$x = \frac{\log 35}{\log 4}$$

$$x \approx 2.5646$$

$$\{2.5646\}$$

If you have a variable exponent but cannot make like bases, take the log or ln of both sides.

$$\log_4 35 = x$$

$$\frac{\log 35}{\log 4} = x$$

e)  $e^x = 15$

$$\ln e^x = \ln 15$$

$$x = \ln 15$$

$$x \approx 2.7081$$

$$\{2.7081\}$$

f)  $\frac{2(3^x)}{2} = \frac{58}{2}$

$$3^x = 29$$

$$\log 3^x = \log 29$$

$$x \log 3 = \log 29$$

$$x = \frac{\log 29}{\log 3}$$

$$x \approx 3.065$$

g)  $3(4^{2m-5}) - 5 = 17$

$$\frac{3(4^{2m-5})}{3} = \frac{22}{3}$$

$$4^{2m-5} = \frac{22}{3}$$

$$\log 4^{2m-5} = \log \left(\frac{22}{3}\right)$$

$$2m-5 = \frac{\log(22/3)}{\log 4}$$

$$2m = \log_4(22/3) + 5$$

$$\{ 3.065 \}$$

$$\log 4$$

$$2m = 6.4372$$

$$m = 3.2186$$

$$\{ 3.2186 \}$$

Factor, ZPP, take log or ln

$$h) 2e^{2x} - 5e^x - 12 = 0$$

$$\begin{array}{r|l} -5e^x & -24 \\ 2e^x - 8e^x & 3 \cdot -8 \end{array}$$

$$(2e^{2x} + 3e^x)(-8e^x - 12) = 0$$

$$e^x(2e^x + 3) - 4(2e^x + 3) = 0$$

$$(e^x - 4)(2e^x + 3) = 0$$

$$e^x - 4 = 0$$

$$e^x = 4$$

$$\ln e^x = \ln 4$$

$$x = 1.3863$$

$$2e^x + 3 = 0$$

$$2e^x = -3$$

$$e^x = -3/2$$

$$\ln e^x = \ln -3/2 \quad x > 0$$

$$\{ 1.3863 \}$$

$$j) e^{2x} - 3e^x + 2 = 0$$

\*Must ✓ your solutions \*

$$(e^{2x} - 2e^x)(-e^x + 2) = 0$$

$$e^x(e^x - 2) - 1(e^x - 2) = 0$$

$$(e^x - 1)(e^x - 2) = 0$$

$$e^x - 1 = 0$$

$$e^x = 1$$

$$\ln e^x = \ln 1$$

$$x = 0$$

$$\{ 0, .6931 \}$$

$$e^x - 2 = 0$$

$$e^x = 2$$

$$\ln e^x = \ln 2$$

$$x = \ln 2$$

$$x = .6931$$

Ex. 2 Solve

$$a) \log x = 4$$

$$b) \ln x = 4$$

$$c) \log 4 = x$$

$$a) \log_5 x = 4$$

$$b) \ln x = 4$$

$$c) \log_6 4 = x$$

$$d) \ln(2x-5) = \ln(7x+27)$$

✓ your solutions!

$$e) \log_4(6x-3) = \log_4(x+17)$$