3.3 Properties of Logarithms

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11:07 AM

Most calculators can only do two types of logs:

common log → log (base 10)   natural log → ln (base e)

If you need to evaluate logs of different bases, use the Change-of-base Formula.

$$\left.\log_a x \longrightarrow \frac{\log x}{\log a}\right\}\text{ base } 10$$

$$\left.\log_a x \longrightarrow \frac{\ln x}{\ln a}\right\}\text{ base } e$$

Ex. 1 Evaluate to 4 decimal places (common log)

a) $\log_6 35$

$\frac{\log 35}{\log 6}$

1.9843

b) $\log_{16} 5$

$\frac{\log 5}{\log 16}$

.5805

c) $\log_{\sqrt{3}} \sqrt{7}$

$\frac{\log \sqrt{7}}{\log \sqrt{3}}$

1.7712

B/C exponential and logarithms are Inverses with base a, the properties of exponential functions have corresponding properties for logarithmic functions.

More Properties of Logarithms

Let a be a positive number such that $a \neq 1$, and let n be a real number. If u and v are positive real numbers, the following properties are true:

	common	natural
1)	$\log_a(uv) \longrightarrow \log_a u + \log_a v$	$\ln(uv) \rightarrow \ln u + \ln v$
2)	$\log_a \frac{u}{v} \longrightarrow \log_a u - \log_a v$	$\ln \frac{u}{v} \rightarrow \ln u - \ln v$

3) $\log_a u^n \longrightarrow n\log_a u$ $\qquad\qquad$ $\ln u^n \longrightarrow n\ln u$

Ex. 2 Using the properties of logs, rewrite the logs in terms of $\ln 2$ and $\ln 5$.

Think factors of 20!

a) $\ln 20$

$\ln(4\cdot 5)$

$\ln 4 + \ln 5$

$\ln 2^2 + \ln 5$

$2\ln 2 + \ln 5$

b) $\ln\left(\frac{2}{125}\right)$

$\ln 2 - \ln 125$

$\ln 2 - \ln 5^3$

$\ln 2 - 3\ln 5$

Rewrite in terms of $\ln 2$ and $\ln 3$

c) $\ln 6$

$\ln 2 + \ln 3$

d) $\ln\left(\frac{8}{81}\right)$

$\ln 8 - \ln 81$

$\ln 2^3 - \ln 3^4$

$3\ln 2 - 4\ln 3$

Ex. 3 Use properties of logs to VERIFY the given statement:

$-\ln(1/3) = \ln 3$

$-\ln(3^{-1}) = \ln 3$

$-1\cdot -1\ln 3 = \ln 3$

$\ln 3 = \ln 3$

$-\ln(1/3) = \ln 3$

$\ln(1/3)^{-1} = \ln 3$

$\ln(3^{-1})^{-1} = \ln 3$

$\ln 3 = \ln 3$

Ex. 4 Write the expression as a sum or difference of logs; express all powers as factors.

a) $\log_a\left(x^2\sqrt{3x-4}\right) \rightarrow \log_a x^2 + \log_a\sqrt{3x-4}$

$2\log_a x + \log_a(3x-4)^{1/2}$

$2\log_a x + \frac{1}{2}\log_a(3x-4)$

FYI

$\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$\sqrt[3]{x^2} = x^{2/3}$

b) $\log\left(\frac{\sqrt[3]{x+2}}{(x-1)^2}\right) \longrightarrow$

$\log \sqrt[3]{x+2} - \log (x-1)^2$

$\log (x+2)^{1/3} - 2\log (x-1)$

$\frac{1}{3}\log (x+2) - 2\log (x-1)$

c) $\log_m \frac{2x\sqrt{x+1}}{(x-3)^3} \longrightarrow$

$\log_m 2x\sqrt{x+1} - \log_m (x-3)^3$

$(\log_m 2x + \log_m \sqrt{x+1}) - 3\log_m (x-3)$

$[(\log_m 2 + \log_m x) + \log_m (x+1)^{1/2}] - 3\log_m (x-3)$

$[(\log_m 2 + \log_m x) + \frac{1}{2}\log_m (x+1)] - 3\log_m (x-3)$

d) $\ln \frac{\sqrt{4x-7}}{x\sqrt[3]{y}} \longrightarrow$

$\ln\sqrt{4x-7} - \ln x\sqrt[3]{y}$

$\ln (4x-7)^{1/2} - (\ln x + \ln \sqrt[3]{y})$

$\frac{1}{2}\ln(4x-7) - (\ln x + \ln y^{1/3})$

$\frac{1}{2}\ln(4x-7) - (\ln x + \frac{1}{3}\ln y)$

Ex. 5 Write each expression as a single logarithm

a) $2\log_a 6 + 5\log_a 4 \longrightarrow$

$\log_a 6^2 + \log_a 4^5$

$\log_a (6^2 \cdot 4^5)$

$\log_a (36 \cdot 1024)$

$\log_a 36{,}864$

b) $\frac{2}{3}\log 27 - \log (2^3 - 4) \longrightarrow$

$\log 27^{2/3} - \log (4)$

$\log 9 - \log 4$

$\log \frac{9}{4}$

c) $4[\ln(x+3) + 3\ln(x+7) - 4\ln(x-8)]$

$4[\ln(x+3) + \ln(x+7)^3 - \ln(x-8)^4]$

$4[\ln(x+3)(x+7)^3 - \ln(x-8)^4]$

$4\left[\ln \frac{(x+3)(x+7)^3}{(x-8)^4}\right]$

$$\ln\left[\frac{(x+3)(x+7)^3}{(x-8)^4}\right]^4$$

$$\ln \frac{(x+3)^4(x+7)^{12}}{(x-8)^{16}}$$