# 3.3 Properties of Logarithms

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11:07 AM

Most calculators can only do two types of logs:

common log → log (base 10)

natural log → ln (base e)

If you need to evaluate logs of different bases, use the Change-of-base Formula.

$$\log_a x \longrightarrow \frac{\log x}{\log a} \quad \} \text{ base } 10$$

$$\log_a x \longrightarrow \frac{\ln x}{\ln a} \quad \} \text{ base } e$$

**Ex. 1** Evaluate to 4 decimal places (common log)

a) $\log_6 35$

$\frac{\log 35}{\log 6}$

1.9843

b) $\log_{16} 5$

$\frac{\log 5}{\log 16}$

.5805

c) $\log_{\sqrt{3}} \sqrt{7}$

$\frac{\log \sqrt{7}}{\log \sqrt{3}}$

1.7712

B/C exponential and logarithms are Inverses with base $a$, the properties of exponential functions have corresponding properties for logarithmic functions.

## More Properties of Logarithms

Let $a$ be a positive number such that $a \neq 1$, and let $n$ be a real number. If $u$ and $v$ are positive real numbers, the following properties are true:

| | Common | Natural |
|---|---|---|
| 1) | $\log_a(uv) \longrightarrow \log_a u + \log_a v$ | $\ln(uv) \rightarrow \ln u + \ln v$ |
| 2) | $\log_a \frac{u}{v} \longrightarrow \log_a u - \log_a v$ | $\ln \frac{u}{v} \rightarrow \ln u - \ln v$ |

3) $\log_a u^n \longrightarrow n\log_a u$ $\qquad\qquad$ $\ln u^n \longrightarrow n\ln u$

Ex. 2 Using the properties of logs, rewrite the logs in terms of $\ln 2$ and $\ln 5$.

think factors of 20!

a) $\ln 20$

$\ln(4\cdot 5)$

$\ln 4 + \ln 5$

$\ln 2^2 + \ln 5$

$2\ln 2 + \ln 5$

b) $\ln\left(\frac{2}{125}\right)$

$\ln 2 - \ln 125$

$\ln 2 - \ln 5^3$

$\ln 2 - 3\ln 5$

Rewrite in terms of $\ln 2$ and $\ln 3$

c) $\ln 6$

$\ln 2 + \ln 3$

d) $\ln\left(\frac{8}{81}\right)$

$\ln 8 - \ln 81$

$\ln 2^3 - \ln 3^4$

$3\ln 2 - 4\ln 3$

Ex. 3 Use properties of logs to VERIFY the given statement:

$-\ln(\frac{1}{3}) = \ln 3$

$-\ln(3^{-1}) = \ln 3$

$-1\cdot -1\ln 3 = \ln 3$

$\ln 3 = \ln 3$

$-\ln(\frac{1}{3}) = \ln 3$

$\ln(\frac{1}{3})^{-1} = \ln 3$

$\ln(3^{-1})^{-1} = \ln 3$

$\ln 3 = \ln 3$

Ex. 4 Write the expression as a sum or difference of logs; express all powers as factors.

c) $\log_a(x^2\sqrt{3x-4}) \rightarrow \log_a x^2 + \log_a \sqrt{3x-4}$

$2\log_a x + \log_a (3x-4)^{1/2}$

$2\log_a x + \frac{1}{2}\log_a(3x-4)$

FYI

$\sqrt{x} = x^{1/2}$

$\sqrt[3]{x} = x^{1/3}$

$\sqrt[3]{x^2} = x^{2/3}$

b) $\log\left(\frac{\sqrt[3]{x+2}}{(x-1)^2}\right) \rightarrow$

c) $\log_m \frac{2x\sqrt{x+1}}{(x-3)^3} \rightarrow$