

## 2.7 Graphs of Rational Functions

### Guidelines for Graphing Rational Functions

Let  $f(x) = \frac{N(x)}{D(x)}$  where  $N(x)$  and  $D(x)$  are polynomials with no common factors;  $D(x)$  not equal to 0.

1. Find y-intercept and plot: evaluate  $f(0)$ . (Plug 0 in for  $x$  and simplify)
2. Find x-intercepts: set numerator equal to 0 and solve for  $x$ .
3. Find points of discontinuity (holes): factor  $N(x)$  and  $D(x)$  and cancel out. (Canceled out factors set = 0)
4. Find vertical asymptotes: set denominator equal to 0. (Don't use any canceled out factors)
5. Find horizontal asymptotes or slant: compare degree of numerator and denominator.
6. Plot at least 1 point between and 1 point beyond each  $x$  intercept.

**Ex. 1** Sketch the graph of each rational function.

a)  $f(x) = \frac{4}{x+3}$

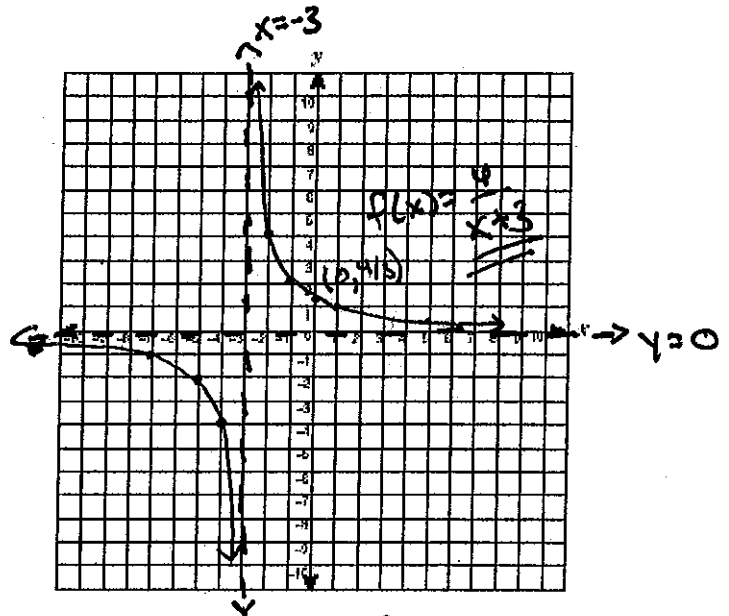
① y-int  
 $y = \frac{4}{0+3}$   
 $y = \frac{4}{3} \rightarrow (0, \frac{4}{3})$

② x-int  
 $0 = 4$   
(NONE)

⑤ HA:  
 $0 < 1$   
 $y = 0$

④ VA  
 $x+3=0$   
 $x = -3$

③ Holes  
NONE



b)  $f(x) = \frac{3x+5}{x}$

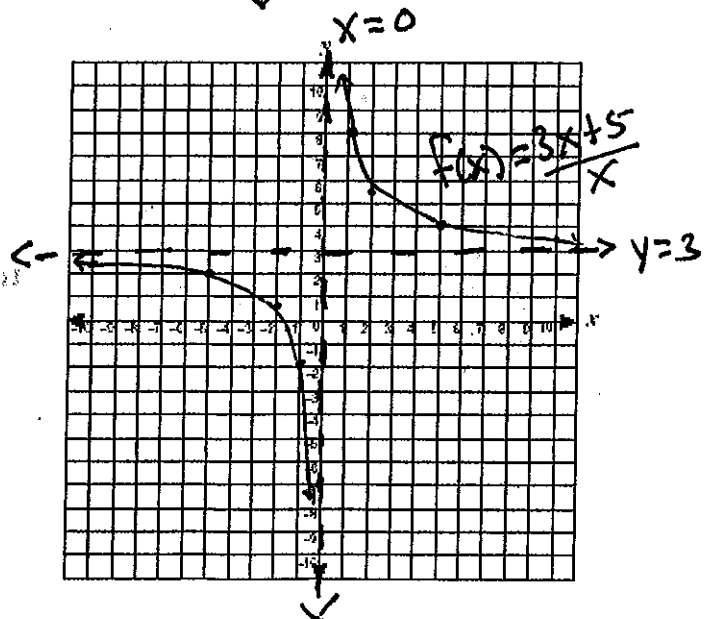
y-int  
 $f(0) = \frac{3(0)+5}{0}$   
 undefined  
(NONE)

x-int  
 $0 = 3x+5$   
 $x = -5/3$   
 $(-5/3, 0)$

Vert. A  
 $x = 0$

Horizontal  
 $1 = 1$   
 $y = 3$

Holes  
NONE



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c)  $f(x) = \frac{x^2 - x - 2}{x + 2}$

y-int  
 $f(0) = \frac{(0)^2 - (0) - 2}{0 + 2}$   
 $f(0) = -1$   
 $(0, -1)$

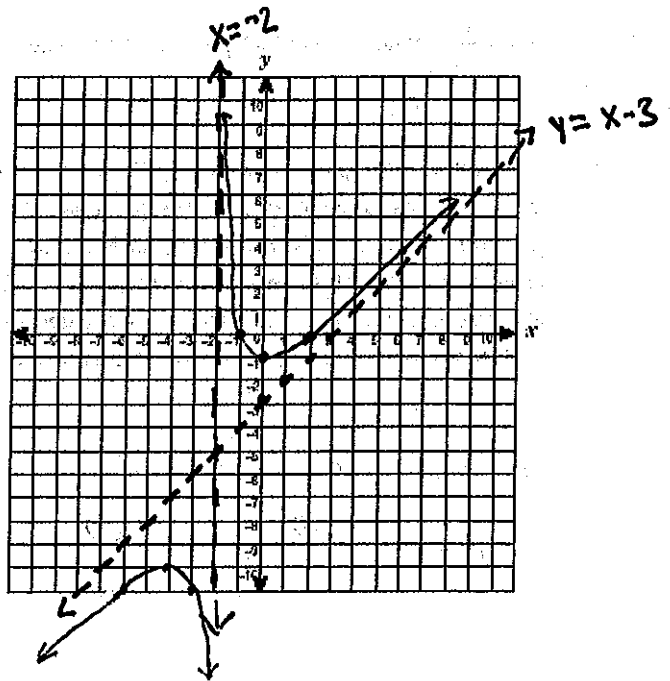
x-int  
 $0 = x^2 - x - 2$   
 $0 = (x - 2)(x + 1)$   
 $x = 2, x = -1$   
 $(2, 0) \quad (-1, 0)$

VA  
 $\frac{(x - 2)(x + 1)}{x + 2}$   
 $x + 2 = 0$   
 $x = -2$

HA  
 $2 > 1$   
NONE

Holes  
 None

Oblique  
 $-2 \overline{) 1 \ -1 \ -2}$   
 $\underline{-2 \ 6}$   
 $1 \ -3 \ 6$   
 $Y = x - 3$



d)  $f(x) = \frac{x^2 + 3x - 4}{x + 2}$

y-int  
 $f(0) = \frac{(0)^2 + 3(0) - 4}{0 + 2}$   
 $f(0) = -2$   
 $(0, -2)$

x-int  
 $0 = (x + 4)(x - 1)$   
 $x = -4 \quad x = 1$   
 $(-4, 0) \quad (1, 0)$

VA  
 $x + 2 = 0$   
 $x = -2$

HA  
 $2 > 1$   
NONE

Oblique  
 $-2 \overline{) 1 \ 3 \ -4}$   
 $\underline{-2 \ -2}$   
 $1 \ 1 \ -6$   
 $Y = x + 1$

