2.6 Rational Functions and Asymptote

Rational Function: (Has a variable in the denominator!)

Can be written in the form $f(x) = \frac{N(x)}{D(x)}$; $D(x) \neq 0$ $\left.\begin{matrix} N(x) \\ D(x) \end{matrix}\right\}$ These are polynomials!

Ex. 1: Simplify each rational expression and state the restrictions on the denominator.

a) $\frac{2x+8}{x^2-16}$

Factoring! $\frac{2(x+4)}{(x+4)(x-4)}$

$\frac{2}{x-4}$; $x \neq \pm 4$

b) $\frac{3x^2y^5}{6xy^2}$

$\frac{1}{2}xy^3$

$\frac{xy^3}{2}$; $x \neq 0$, $y \neq 0$

c) $\frac{y^2-16}{y^2+7y+12}$

Factoring $\frac{(y-4)(y+4)}{(y+3)(y+4)} \Rightarrow \frac{y-4}{y+3}$; $y \neq -3$, $y \neq -4$

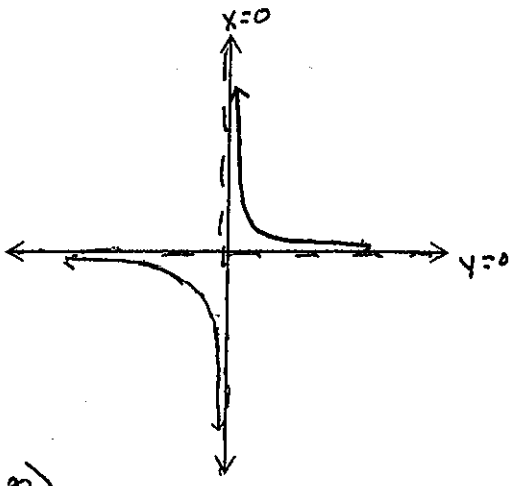
For the function $f(x) = \frac{1}{x}$, complete the tables below to:

a) State the domain

b) Look at the end behavior

x	-1	-.5	-.1	-.01	-.001	$\rightarrow 0$
f(x)	-1	-2	-10	-100	-1000	$-\infty$

x	$0 \leftarrow$	.001	.01	.1	.5	1
f(x)	∞	1000	100	10	2	1



Domain: All real numbers except $x=0$; $(-\infty, 0)$ $(0, \infty)$

Graph will never cross the y-axis; $x=0$ is a vertical asymptote.

Graph will never cross the x-axis; $y=0$ is a horizontal asymptote.

Asymptotes

Are lines (boundaries) that a graph approaches as the ~~as the~~ graph approaches infinity. Asymptotes are represented as dashed lines (if not an axis).

In the above example, the behavior of f(x) = $\frac{1}{x}$ near x = 0: as $x \rightarrow 0^+$, $f(x) \rightarrow \infty$. (f(x) increases without bound as x approaches 0 from the right) and $x \rightarrow 0^-$, $f(x) \rightarrow -\infty$ (f(x) decreases without bound as x approaches 0 from the left) So, x = 0 is a vertical asymptote of the graph of f(x).

The graph also has a horizontal asymptote y = 0

- $f(x) \rightarrow 0$ as $x \rightarrow -\infty$ (f(x) approaches 0^- as x decreases without bound)
- $f(x) \rightarrow 0$ as $x \rightarrow \infty$ (f(x) approaches 0^- as x increases without bound)

2.6 Rational Functions and Asymptote

Vertical Asymptote (x=a)

Is the line x = **a** of the graph *f* if : $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$

Horizontal Asymptote (y=b)

Is the line y = **b** of the graph *f* if : $f(x) \to b$ or $x \to \infty$ or $x \to -\infty$

Finding Asymptotes

Vertical Asymptotes (VA):

- Find the restrictions of the denominator (values x cannot equal)
- Factor and Simplify the rational expression.
- Cancel out any common factors (the canceled out factors are POINTS OF DICONTINUITIES or Holes in the graph).
- Set the remaining factors in the denominator = 0; solve for x.

Ex. 2: Find the VA and any holes for the function.

a)

b)

c)

This means there will be a hole in the graph at _____.

To find the ordered pair for the hole, substitute _____ into your simplified function.

Horizontal Asymptotes (HA):

The graph of *f* has _____horizontal asymptote determined by comparing the degrees of the numerator and the denominator.

1. If the degree of the numerator the degree of the denominator, the line _____ is the HA.