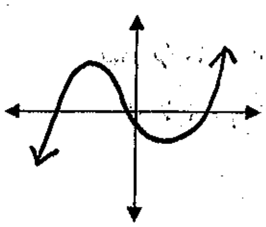
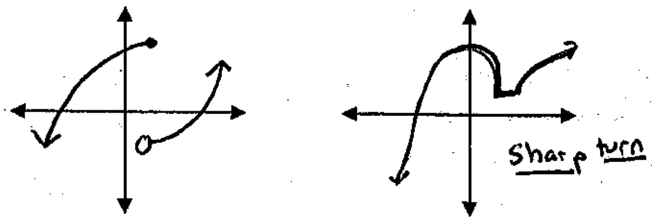
2.2 Polynomial Functions of Higher Degree

Graph of a Polynomial Function is continuous; this means the graph has no holes, breaks, gaps, or sharp turns. Graphs are smooth and continuous.

Polynomial function



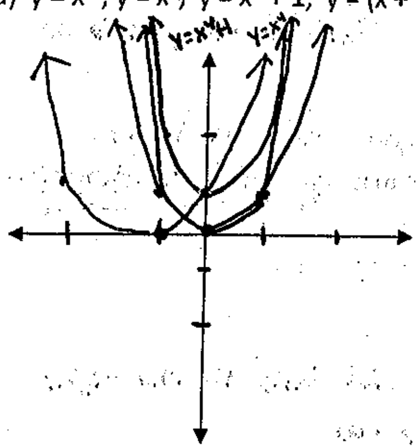
Non-polynomial function



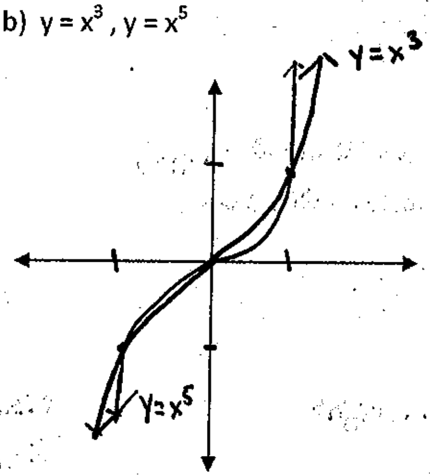
Has breaks; not continuous.

Ex. 1 Graph using a calculator; put the even degree functions on one graph and the odd degree on another.

a) $y = x^2$, $y = x^4$, $y = x^4 + 1$, $y = (x+1)^4$



b) $y = x^3$, $y = x^5$



Leading Coefficient Test $\quad f(x) = a_nx^n + \cdots + a_1x + a_0$

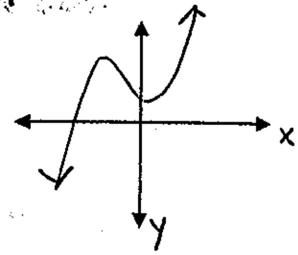
This test allows us to determine a functions "END BEHAVIOR" by looking at its degree (even or odd) and the leading coefficient.

a_nx^n ← highest degree

LC

When n is odd:

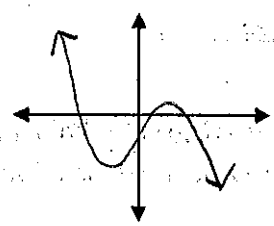
Leading coefficient is positive ($a_n > 0$) the graph falls to the left and rises to the right.



$f(x) \to -\infty,\ x \to -\infty$
$f(x) \to \infty,\ x \to \infty$

n is odd

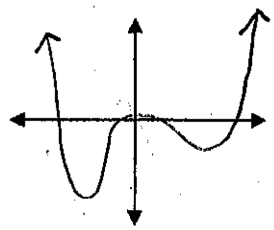
Leading coefficient is negative ($a_n < 0$)
the graph rises to the left and
falls to the right.



$f(x) \to \infty$, $x \to -\infty$
$f(x) \to -\infty$, $x \to \infty$

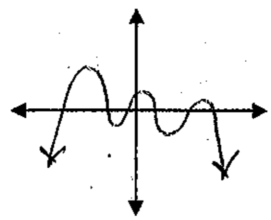
When n is even:

Leading coefficient is positive ($a_n > 0$)
the graph rises to the left and
rises to the right.



$f(x) \to \infty$, $x \to -\infty$
$f(x) \to \infty$, $x \to \infty$

Leading coefficient is negative ($a_n < 0$)
the graph falls to the left and
falls to the right.



$f(x) \to -\infty$, $x \to -\infty$
$f(x) \to -\infty$, $x \to \infty$

* Odd degrees have no limited ranges, arrows go in opposite direction.

* Even degrees have a limited range; arrows go in same direction.

Ex. 2 Describe the end behavior of the graph of each function. (Don't graph, use the LCT.)

(a) $y = x^4 + 2x^2 - 3x$

rises to the left and right

$f(x) \to \infty$, $x \to -\infty$
$f(x) \to \infty$, $x \to \infty$

(b) $y = -x^5 + 3x^3 - x$

rises to the left and falls to the right

$f(x) \to \infty$, $x \to -\infty$
$f(x) \to -\infty$, $x \to \infty$

(c) $y = 2x^3 - 3x^2 + 5$

falls to the left and rises to the right

$f(x) \to -\infty$, $x \to -\infty$
$f(x) \to \infty$, $x \to \infty$

(d) $y = -5x^4 - 6$

falls to the left and right

$f(x) \to -\infty$, $x \to -\infty$
$f(x) \to -\infty$, $x \to \infty$

Polynomial Functions:

1. The graph of *f* has **at most** ***n*** zeros.
2. The function of *f* has **at most** ***n* – 1** relative maximum(s) or minimum(s).

Zero: where the graph crosses the x-axis; $f(x) = 0$
$y = 0$

Real Zeros of Polynomial Functions

If *f* is a polynomial function and *a* is a real number, then:

1. x = a is a zero of the function "f".
2. x = a is a solution of the polynomial eqn. $f(x) = 0$
3. (x – a) is a <u>factor</u> of the polynomial $f(x)$
4. (a, 0) is an x-intercept of the graph of "f"

"a is not the LC."

$y = (x-2)^2$
$\sqrt{0} = \sqrt{(x-2)^2}$
$0 = x - 2$
$x = 2$ $(x-2)$
$(2, 0)$

Ex. 3 Find the zeros of a polynomial function.

SET = 0

(a) $f(x) = x^3 - x^2 - 2x$

$x^3 - x^2 - 2x = 0$
$x(x^2 - x - 2) = 0$
$x(x+1)(x-2) = 0$
$x = 0$ $x+1=0$ $x-2=0$
$x = -1$ $x = 2$
$\{-1, 0, 2\}$ / $(-1,0)\ (0,0)\ (2,0)$

(b) $f(x) = 2x^3+11x^2+12x$

$2x^3+11x^2+12x = 0$
$x(2x^2+11x+12) = 0$
$x(2x^2+3x)(8x+12) = 0$
$x[x(2x+3)+4(2x+3)] = 0$
$x(x+4)(2x+3) = 0$
$x = 0$ $x+4=0$ $2x+3=0$
$x = -4$ $x = -3/2$
$\{-4, -3/2, 0\}$

11x	24
	1·24
	2·12
3x+8x	3·8

(c) $f(x) = x^4 - 8x^2 + 15$

$(x^4 - 3x^2)(-5x^2+15)$
$x^2(x^2-3) - 5(x^2-3)$
$(x^2-5)(x^2-3) = 0$ ⟶

$-8x^2$	15
	-1·-15
$-3x^2-5x^2$	-3·-5

$x^2 - 5 = 0$ $x^2 - 3 = 0$
$x^2 = 5$ $x^2 = 3$
$x = \pm\sqrt{5}$ $x = \pm\sqrt{3}$ $\{\pm\sqrt{3}, \pm\sqrt{5}\}$

If you are given the zeros of a polynomial function, you can work backwards and find the function itself.

Ex. 4 Write a polynomial function given zeros:

(a) {-2, -1, 1, 2}

$x = -2$ $x = -1$ $x = 1$ $x = 2$
$x+2=0$ $x+1=0$ $x-1=0$ $x-2=0$
$(x+2)(x+1)(x-1)(x-2) = 0$
$(x^2-4)(x^2-1) = 0$

	x^2	-4
x^2	x^4	$-4x^2$
-1	$-x^2$	$+4$

$\boxed{f(x) = x^4 - 5x^2 + 4}$

(b) {3, -3/4, 1}

$x = 3 \quad x = -3/4 \quad x = 1$

$x - 3 = 0 \quad 4x = -3 \quad x - 1 = 0$

$4x + 3 = 0$

$(x-3)(4x+3)(x-1) = 0$

$(x^2 - 4x + 3)(4x+3) = 0$

$f(x) = 4x^3 - 13x^2 + 9$

(c) {5, 2 - √7, 2 + √7}

$x = 5 \quad x = 2 - \sqrt{7} \quad x = 2 + \sqrt{7}$

$(x-5) \quad (x - 2 + \sqrt{7}) \quad (x - 2 - \sqrt{7})$

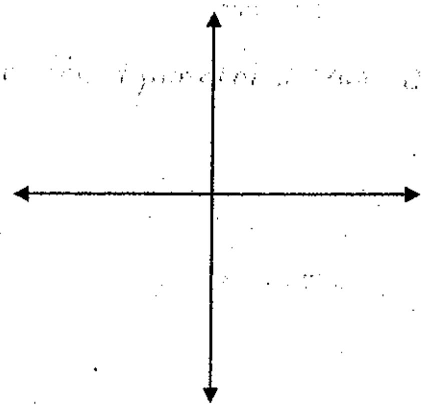
	x	-2	$-\sqrt{7}$
x	x^2	$-2x$	$-x\sqrt{7}$
-2	$-2x$	4	$2\sqrt{7}$
$\sqrt{7}$	$x\sqrt{7}$	$-2\sqrt{7}$	-7

$(x-5)(x^2 - 4x - 3) = 0$

$\boxed{f(x) = x^3 - 9x^2 + 17x + 15}$

Ex. 5 **Find all the zeros and relative extrema (max and min); then sketch the graph.**

(a) $y = -2x^4 + 2x^2$



Multiplicity is