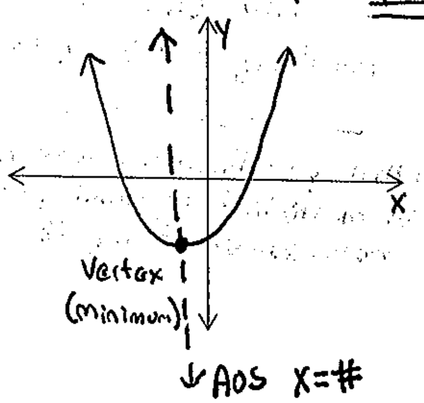
2.1 Quadratic Functions

Quadratic Function: $f(x) = ax^2 + bx + c,\ a \neq 0$

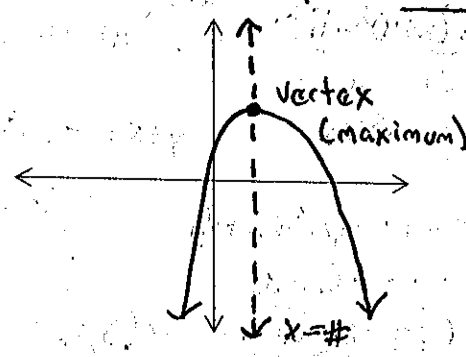
Basic Review:

- ✓ Graphs of this function are called **PARABOLAS**
- ✓ Symmetric with respect to the **Axis of Symmetry**; a vertical line (the **x-coordinate** of the vertex).
  $x = \#$ or AOS $(x) = \frac{-b}{2a}$
- ✓ c is the y-intercept; (0, c)
- ✓ x-intercept(s) are the **ZEROs** of the function; *aka the solution* ***(roots)*** *to the equation when set equal to 0.*
- ✓ The vertex contains the maximum or minimum value of the function (the **y-coordinate** of the vertex).

$a > 0$ Opens upward ↑



$a < 0$ Opens downward ↓



Vertex form (Standard form)

$f(x) = a(x-h)^2 + k$ vertex is (h, k)

↳ * notice the "−h" is now "h" in the vertex.

Ex. 1 Describe the transformations from the parent function of $y = x^2$.

(a) $f(x) = 6x^2$

vertical stretch

(b) $y = -x^2 + 6$

reflects over x-axis, shifts up 6 units

(c) $y = \frac{1}{5}x^2 - 5$

vertical compression, shifts down 5 units

(d) $y = (x-3)^2 + 4$

shifts right 3 units up 4 units.

A quadratic function can be written in Standard Form (vertex); you need to **"complete the square"** on the form

$f(x) = ax^2 + bx + c$ -------> $f(x) = a(x-h)^2 + k$

Complete the square steps

① Move "c" to the left; get ax^2 and bx alone.

② Divide "b" by 2

③ Take step 2's result and square

④ Take step 3's result and add to both sides.

⑤ You have a perfect square on the x^2+bx side, factor it into $(a \pm b)^2$

⑥ Bring "c" back to the right.

Ex. 2: Write the standard form (vertex form) and name the vertex.

(a) $y = x^2 + 8x + 5$

$-5 \qquad -5$

$y-5 = x^2+8x \qquad \frac{8}{2} \to 4 \to (4)^2 \to 16$

$y-5+16 = x^2+8x+16$

$y+11 = (x+4)^2$ This is the factored form.

$y=(x+4)^2-11 \longrightarrow$ Vertex $(-4,-11)$

(b) $y = -x^2 + 6x - 8$

*need to factor out -1!

$y+8 = -x^2+6x \qquad y+8 = -(x^2-6x+\_)$

* what you factored out has to be multiplied to what you add to the left side of eqn *

$y+8-9 = -(x^2-6x+9) \qquad -\frac{6}{2} \to -3 \to (-3)^2 = 9$

$-1 \cdot 9 = -9$ Left side

$y-1 = -(x-3)^2$

$y = -(x-3)^2+1 \quad V(3,1)$

(c) $y = -2x^2 + 12x - 3$

$y+3 = -2(x^2-6x+\_) \qquad -\frac{6}{2} = -3 \to (-3)^2 \to 9$

$y+3-18 = -2(x^2-6x+9) \qquad -2 \cdot 9 = -18$

$y-15 = -2(x-3)^2 \longrightarrow y = -2(x-3)^2+15 \quad V(3,15)$

Ex. 3: Write the equation of the parabola in standard form.

(a) vertex (1, 2) and passing through (3, -6)

$y = a(x-h)^2+k$ substitute in the vertex for "h" and "k" and the other coordinate given for "x" and "y". Solve for "a"

$-6 = a(3-1)^2+2$

$-6 = a(2)^2+2$

$-6 = 4a+2 \rightarrow -8 = 4a \rightarrow a = -2$

$y = -2(x-1)^2+2$

(b) vertex (-2, 5) and passes through (0, 9)

$y = a(x-h)^2+k$

$9 = a(0+2)^2+5$

$9 = a(2)^2+5$

$4 = 4a$

$a = 1$

$y = (x+2)^2+5$

Applications of Quadratic Functions (Maximum/Minimum)

Many applications (word problems) involve finding the MAX or MIN value of a quadratic function. There are several ways to find the MAX or MIN value of a quadratic function:

1) Graphing calculator MAX/MIN function
2) Finding the AOS and then substituting into the function that value to get the MAX/MIN
3) Writing the equation in STANDARD (Vertex) FORM

Ex. 4: Solve each application.

a) A baseball is hit 3 feet above the ground at a velocity of 100 feet per second and at an angle of 45 degrees with respect to ground level. The path of the baseball is given by the function $f(x) = -0.0032x^2 + x + 3$, where *f(x)* is the height of the baseball (feet) and *x* is the distance from home plate (feet). **What is the maximum height the baseball will reach?**

* Show graphically / algebraically

$\text{AOS} = \frac{-b}{2a}$

$f(x) = -0.0032x^2 + x + 3$

$\text{AOS} = \frac{-(1)}{2(-0.0032)}$ $\quad x = 156.25$

$f(156.25) = -0.0032(156.25)^2 + 156.25 + 3$

$f(156.25) = 81.125$ ft max height

b) A textile company has a daily production costs of $C(x) = 10{,}000 - 110x + 0.45x^2$, where C is the total cost (\$) and the x is the number of units produced. **How many units should be produced each day to yield a minimum cost?**

$C(x) = 10{,}000 - 110x + .45x^2$

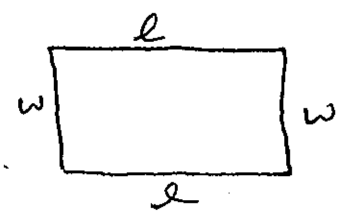
$\text{AOS} = \frac{-(-110)}{2(.45)}$ $\quad x = 122.22$

$f(122.\overline{22}) = 10{,}000 - 110(122.22) + .45(122.22)^2$

$= \$3277.78$

122 units gives a minimum cost of \$3277.78

c) Mr. Garvey has a section of land that he wants to fence in to make a paddock for his horses. He has 400 feet of fencing to accomplish this task. What is the maximum area he can have with this amount of fencing material? What will the dimensions be of the fenced in area?



$P = 2l + 2w$ $\qquad A = lw$

$400 = 2l + 2w$ $\qquad A = l(200 - l)$

$400 - 2l = 2w$ $\qquad 0 = 200l - l^2$

$200 - l = w$ $\qquad a = -1 \quad b = 200$

$\text{AOS} = \frac{-(200)}{2(-1)}$

$= 100$ ft

Maximum area is 10,000 ft^2 and dimensions are 100' x 100'

$w = 200 - l$
$= 200 - 100$
$= 100$

$A = 100(200 - 100)$
$= 100(100)$
$A = 10{,}000$ ft^2