12.4 Limits at Infinity and of Sequences

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Defn. of Limits at Infinity

If f is a function and L_1 and L_2 are real numbers, then the statement

$$\lim_{x \to -\infty} f(x) = L_1 \quad \text{and} \quad \lim_{x \to \infty} f(x) = L_2$$

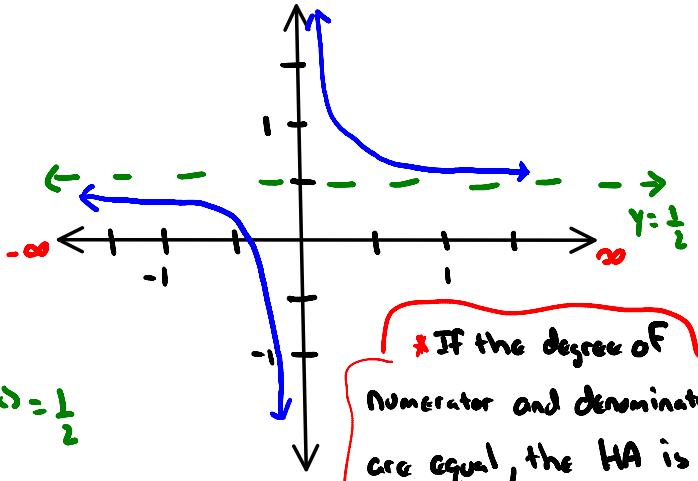
denotes the limits at infinity.

ex: $f(x) = \frac{x+1}{2x}$

* Horizontal Asymptote

$y = \frac{1}{2}$

$\lim_{x \to -\infty} f(x) = \frac{1}{2}$ $\lim_{x \to \infty} f(x) = \frac{1}{2}$



* If the degree of numerator and denominator are equal, the HA is the ratio of the LC's.

Limits at Infinity

If r is a positive real number, then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

If x^r is defined when $x < 0$, then

$$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

Ex. 1 Find the limit

a) $\lim$ $\lim$

a) $\lim_{x\to\infty} \frac{1}{x^5}$ $\lim_{x\to-\infty} \frac{1}{x^5}$

$\frac{1}{(1)^5}=1$ $\frac{1}{(2)^5}=\frac{1}{32}$ $\frac{1}{(-1)^5}=-1$ $\frac{1}{(-2)^5}=\frac{-1}{32}$

$\frac{1}{(3)^5}=\frac{1}{243}$ $\frac{1}{(-3)^5}=\frac{-1}{243}$

$\lim_{x\to\infty} \frac{1}{x^5}=0$ $\lim_{x\to-\infty} \frac{1}{x^5}=0$

The limit is 0.

b) $\lim_{x\to\infty} \left(4-\frac{3}{x^2}\right)$

$\lim_{x\to\infty} 4 - \lim_{x\to\infty} \frac{3}{x^2}$ $\frac{3}{(1)^2}=3$ $\frac{3}{(2)^2}=\frac{3}{4}$ $\frac{3}{(3)^2}=\frac{3}{9}$

$4-0$ $\frac{3}{(4)^2}=\frac{3}{16}$ $\frac{3}{(5)^2}=\frac{3}{25}$

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The limit is 4

Limits at Infinity for Rational Functions

3 cases for the limit as x approaches $\pm\infty$.

A) degree of numerator < denominator, then the limit will be 0.

ex: $f(x)=\frac{x^2+1}{x^3+3}$ $2<3$

$\lim_{x\to\pm\infty} \frac{x^2+1}{x^3+3}=0$

B) degree of numerator = denominator, then the ratio

of the leading coefficients will be the limit.

ex: $f(x) = \frac{3x^4}{4x^4-1}$ $4=4$

$\lim_{x\to\pm\infty} \frac{3x^4}{4x^4-1} = \frac{3}{4}$

c) degree of numerator > denominator, then the limit does not exist.

ex: $f(x) = \frac{x^5}{x^2-25}$ 5>2

$\lim_{x\to\pm\infty} \frac{x^5}{x^2-25}$ = does not exist

Ex. 2 Find the limit as $x \to \infty$.

a) $f(x) = \frac{x+5}{x^2-1}$ b) $f(x) = \frac{-5x^4+3}{6x^3+1}$ c) $f(x) = \frac{3x^2+5}{7x^2-2}$

$\lim_{x\to\infty} f(x) = 0$ $\lim_{x\to\infty} f(x)$: does not exist $\lim_{x\to\infty} f(x) = \frac{3}{7}$

Limits of Sequences (same properties as limits of functions)

consider the sequence $a_n = .5^n$ $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$

As n increases, the terms of the sequence get closer to 0. The sequence is said to Converge to 0.

If a sequence does not converge, it is said to diverge. the limit does not exist.

ex: $1,-1,1,-1,1,\ldots$ $a_n=(-1)^{n-1}$

Ex. 3 Find the limit of the sequence

a) $a_n = \frac{2n^2+1}{n+4}$

$\lim_{x\to\infty} \frac{2n^2+1}{n+4}$ does not exist b/c the sequence diverges

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b) $a_n = \frac{2n+1}{4n^2}$

$\lim_{x\to\infty} \frac{2n+1}{4n^2} = 0$, b/c converges to 0.

c) $a_n = \frac{2n^2+1}{4n^2}$

$\lim_{x\to\infty} \frac{2n^2+1}{4n^2} = \frac{1}{2}$; converges.

Ex. 4

You are manufactoring a product that costs $\$0.75$ per unit to produce. Your initial investment is $\$6000$, which implies that the total cost of producing "x" units is $C = .75x + 6000$. The average cost per unit is:

$$\bar{C} = \frac{C}{x} = \frac{0.75x + 6000}{x}$$

$\bar{C}$ is Average
C is function
x is # of units

Find the average cost per unit when when:

a) $x = 1000$ $\quad \bar{C} = \$6.75$

b) $x = 10{,}000$ $\quad \bar{C} = \$1.35$

c) $x = 100{,}000$ $\quad \bar{C} = \$0.81$

d) what is the limit of the average cost per unit when

$x \to \infty$?

$$\lim_{x \to \infty} \frac{.75x + 6000}{x}$$

1 : 1

$0.75 is the average cost.

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