12.2 Techniques for Evaluating Limits

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11:15 AM

Limits of Polynomial and Rational Functions (nonzero denominators)

$(2x^2+6x+10)$ $\left(\frac{2x+5}{x-1}\right)$

If p is a polynomial function and c is a real number, then

$$\lim_{x \to c} p(x) = p(c)$$

If r is a rational function given by $r = \frac{p(x)}{q(x)}$, and c is a real number such that $q(c) \neq 0$, then

$$\lim_{x \to c} r(x) = r(c) \rightarrow \frac{p(c)}{q(c)} \qquad q(c) \neq 0$$

Above defn. references direct substitution and works for "well behaved" functions.

Ex. 1 Evaluate using direct substitution.

a) $\lim_{x \to 1} \frac{5x^3 - x + 2}{3x+4}$

$$\frac{5(1)^3 - (1) + 2}{3(1)+4}$$

$$\lim_{x \to 1} \frac{5x^3 - x + 2}{3x+4} = \frac{6}{7}$$

b) $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9}$

$\frac{0}{0} \rightarrow$ cannot use direct substitution, b/c at $x=3$, the function is undefined.

$\frac{0}{0}$ has no real meaning and is called the Indeterminate Form, b/c you cannot determine the limit using direct substitution.

If you get $\frac{0}{0}$, you can conclude that the numerator and denominator have a common factor!

Dividing Out Technique

- Rational Function, encounter $\frac{0}{0}$ from direct substitution
- Factor numerator and denominator
- Divide out any common factors
- Try direct substitution again with the new function

- Try direct substitution again with the new function.

Ex. 2 Find the limit of each function.

a) $\lim_{x \to 3} \frac{x^2 - x - 6}{x^2 - 9}$

$\frac{(3)^2 - (3) - 6}{(3)^2 - 9} = \frac{0}{0}$

$\frac{(x+2)(x-3)}{(x+3)(x-3)}$

$\lim_{x \to 3} \frac{x+2}{x+3}$

$\frac{3+2}{3+3}$

$\lim_{x \to 3} \frac{5}{6}$

b) $\lim_{x \to -3} \frac{x^2 + x - 6}{x+3}$

direct sub $= \frac{0}{0}$

$\frac{(x+3)(x-2)}{x+3}$

$\lim_{x \to -3} x - 2$

$-3 - 2$

$\lim_{x \to -3} x - 2 = -5$

c) $\lim_{x \to 1} \frac{x-1}{x^3 - x^2 + x - 1}$ Factor by grouping!

$\frac{x-1}{(x^3 - x^2) + (x-1)} = \frac{x-1}{(x^2+1)(x-1)}$

$x^2(x-1) + 1(x-1)$

$\lim_{x \to 1} \frac{1}{x^2+1} = \frac{1}{2}$

d) $\lim_{x \to 0} \frac{5x - \sin x}{x}$

$\frac{5x}{x} - \frac{\sin x}{x}$

$\frac{\sin x}{x} \approx 1$

$5 - \frac{\sin x}{x}$

$5 - 1$

$\lim_{x \to 0} \frac{5x - \sin x}{x} = 4$

e) $\lim_{x \to 2} \frac{x^3 - 2x^2 + 4x - 8}{x^4 - 2x^3 + x - 2}$

$\frac{(x^3 - 2x^2) + (4x - 8)}{(x^4 - 2x^3) + (x-2)} \rightarrow \frac{x^2(x-2) + 4(x-2)}{x^3(x-2) + 1(x-2)}$

$\lim_{x \to 2} \frac{x^2+4}{x^3+1} = \frac{8}{9}$

f) Find the limit as x approaches 2 on the average rate of change of the function $f(x) = x^2 + 3x$.

Δ delta (change in)

$\lim_{x \to 2} \frac{\Delta y}{\Delta x} \rightarrow \frac{f(x) - f(2)}{x - 2}$

$\frac{(x^2 + 3x) - ((2)^2 + 3(2))}{x - 2}$

$\lim_{x \to 2} \frac{x^2 + 3x - 10}{x - 2} \rightarrow \frac{(x+5)(x-2)}{x-2}$

$$\lim_{x\to 2} \frac{x^2+3x-10}{x-2} \rightarrow \frac{(x+5)\cancel{(x-2)}}{\cancel{x-2}}$$

$$\lim_{x\to 2} x+5 = 7$$

Rationalizing Technique

Rationalize the numerator if $\frac{0}{0}$ is a result of direct substitution. Use this technique if there is a $\sqrt{\quad}$ in numerator.

Must use the conjugate to rationalize!

$$\sqrt{3}+x \cdot \overset{\text{conjugate}}{\sqrt{3}-x} = 3-x^2$$

Ex. 3 Find the limit if it exists.

a) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$

direct sub $\frac{0}{0}$.

do not distribut

$$\frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}$$

	$\sqrt{x+1}$	-1
$\sqrt{x+1}$	$x+1$	$-\sqrt{x+1}$
$+1$	$\sqrt{x+1}$	-1

$x+1-1$

x

$$\lim_{x\to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)}$$

$$\lim_{x\to 0} \frac{1}{\sqrt{x+1}+1} = \frac{1}{2}$$

$$\frac{1}{\sqrt{(0)+1}+1} = \frac{1}{2}$$

$1-x-4 = -x-3$

b) $\lim_{x\to -3} \frac{\sqrt{1-x}-2}{x+3}$

direct sub. $= \frac{0}{0}$

$$\frac{\sqrt{1-x}-2}{x+3} \cdot \frac{\sqrt{1-x}+2}{\sqrt{1-x}+2} \quad \frac{-x-3}{(x+3)(\sqrt{1-x}+2)} = \frac{-1\cancel{(x+3)}}{\cancel{(x+3)}(\sqrt{1-x}+2)}$$

$$\lim_{x\to -3} \frac{-1}{\sqrt{1-x}+2} = -\frac{1}{4}$$

When trying to find the limits of nonalgebraic functions, you may need to use your calculator's graphing ability and/or table.

Ex. 4 Approximate the limit using your calculator.

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a) $\lim_{x\to 0} \frac{\sin x}{x}$

1

b) $\lim_{x\to 0} (1+x)^{1/x}$

2.7

One - Sided Limit

Is a limit that fails to exist b/c a function approaches a different value from the left side of c than it approaches from the right side of c.

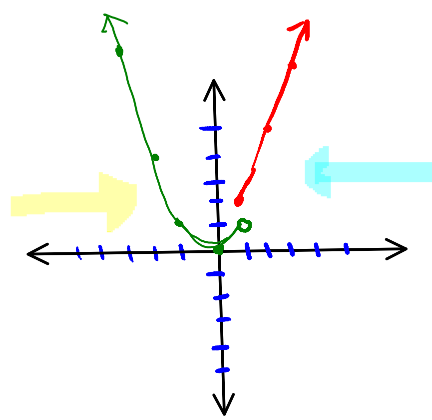
Analyzes behavior of graph from one side:

$\lim_{x\to c^+} f(x) = L$ (from right)

$\lim_{x\to c^-} f(x) = L$ (from left)

Ex. 5 Find the limit if it exists.

a) $f(x) = \begin{cases} x^2, & x < 1 \\ 3x-1, & x \geq 1 \end{cases}$ (from the left) (from the right)



a) $\lim_{x\to 1^-} f(x)$

$(x)^2$

$(1)^2$

$= 1$

b) $\lim_{x\to 1^+} f(x)$

$3x-1$

$3(1)-1$

$= 2$

limit does not exist

b) $f(x) = \begin{cases} 4-x, & x < 1 \\ 4x-x^2, & x > 1 \end{cases}$

1) $\lim_{x\to 1^-} f(x)$

$4-x$

$4-(1)$

3

2) $\lim_{x\to 1^+} f(x)$

$4x-x^2$

$4(1)-(1)^2$

3

Limit is 3

c) $f(x) = \frac{|2x|}{x}$

1) $\lim_{x \to 0^-} f(x)$

$\frac{-(2x)}{x}$

2) $\lim_{x \to 0^+} f(x)$

$\frac{2x}{x}$

Limit Does not exist

A Limit From Calculus

Difference Quotient $\frac{f(x+h)-f(x)}{h}$, $h \neq 0$

For the function $f(x) = x^2 - 1$ find the $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h}$

$f(3+h) = (3+h)^2 - 1$

$h^2 + 6h + 8$

$f(3) = (3)^2 - 1$

8

$\lim_{h \to 0} \frac{(h^2+6h+8)-8}{h} = \frac{h^2+6h}{h}$

$\frac{\cancel{h}(h+6)}{\cancel{h}}$

$\lim_{h \to 0} h+6 = 6$

Ch.1

$h+6$, $h \neq 0$