12.2 Techniques for Evaluating Limits
Wednesday, May 20, 2015
11:15 AM

Limits of Polynomial and Rational Functions (nonzero denominators)
$(2x^2+6x+10)$ $\left(\frac{2x+5}{x-1}\right)$

If p is a polynomial function and c is a real number, then

$$\lim_{x\to c} p(x) = p(c)$$

If r is a rational function given by $r=\frac{p(x)}{q(x)}$, and c is a real number such that $q(c)\neq 0$, then

$$\lim_{x\to c} r(x) = r(c) \rightarrow \frac{p(c)}{q(c)} \qquad q(c)\neq 0$$

Above defn. references direct substitution and works for "well behaved" functions.

Ex. 1 Evaluate using direct substitution.

a) $\lim_{x\to 1} \frac{5x^3-x+2}{3x+4}$

$\frac{5(1)^3-(1)+2}{3(1)+4}$

$\lim_{x\to 1} \frac{5x^3-x+2}{3x+4} = \frac{6}{7}$

b) $\lim_{x\to 3} \frac{x^2-x-6}{x^2-9}$

$\frac{0}{0}\rightarrow$ cannot use direct substitution, b/c at $x=3$, the function is undefined.

$\frac{0}{0}$ has no real meaning and is called the Indeterminate Form, b/c you cannot determine the limit using direct substitution.

If you get $\frac{0}{0}$, you can conclude that the numerator and denominator have a common factor!

Dividing Out Technique
- Rational Function, encounter $\frac{0}{0}$ from direct substitution
- Factor numerator and denominator
- Divide out any common factors

- Divide out any common factors
- Try direct substitution again with the new function.

Ex. 2 Find the limit of each function.

a) $\lim_{x\to 3} \frac{x^2-x-6}{x^2-9}$

$\frac{(3)^2-(3)-6}{(3)^2-9} = \frac{0}{0}$

$\frac{(x+2)(x-3)}{(x+3)(x-3)}$

$\lim_{x\to 3} \frac{x+2}{x+3}$

$\frac{3+2}{3+3}$

$\lim_{x\to 3} \frac{5}{6}$

b) $\lim_{x\to -3} \frac{x^2+x-6}{x+3}$

direct sub $= \frac{0}{0}$

$\frac{(x+3)(x-2)}{x+3}$

$\lim_{x\to -3} x-2$

$-3-2$

$\lim_{x\to -3} x-2 = -5$

c) $\lim_{x\to 1} \frac{x-1}{x^3-x^2+x-1}$ Factor by grouping!

$\frac{x-1}{(x^3-x^2)+(x-1)} = \frac{x-1}{(x^2+1)(x-1)}$

$x^2(x-1)+1(x-1)$

$\lim_{x\to 1} \frac{1}{x^2+1} = \frac{1}{2}$

d)

e) $\lim_{x\to 2} \frac{x^3-2x^2+4x-8}{x^4-2x^3+x-2}$

$\frac{(x^3-2x^2)+(4x-8)}{(x^4-2x^3)+(x-2)} \to \frac{x^2(x-2)+4(x-2)}{x^3(x-2)+1(x-2)}$

$\lim_{x\to 2} \frac{x^2+4}{x^3+1} = \frac{8}{9}$

f) Find the limit as x approaches 2 on the average rate of change of the function $f(x) = x^2+3x$.

Δ delta (change in)

$\lim_{x\to 2} \frac{\Delta y}{\Delta x} \to \frac{f(x)-f(2)}{x-2}$

$(x^2+3x) - ((2)^2+3(2))$

$$\frac{(x^2+3x)-((2)^2+3(2))}{x-2}$$

$$\lim_{x \to 2} \frac{x^2+3x-10}{x-2} \rightarrow \frac{(x+5)\cancel{(x-2)}}{\cancel{x-2}}$$

$$\lim_{x \to 2} x+5 = 7$$