10.8 Polar Equations of Conics

Alt. Defn of a Conic

The locus of a point in a plane which moves so that its distance from a fixed point (focus) is in a <u>constant ratio</u> to its distance from a fixed line (directrix) is a conic.

constant ratio is the <u>eccentricity</u> of the <u>conic</u>

$$e = \frac{c}{a}$$

$e < 1$ your conic is an ellipse

$e = 1$ your conic is a parabola

$e > 1$ your conic is a hyperbola

$r = 2$ Circle $\qquad \theta = \frac{\pi}{6}$ Line

Polar equations of conics

"d" = ep

A) $r = \dfrac{d}{1 \pm e\cos\theta}$ <u>vertical directrix</u>

B) $r = \dfrac{d}{1 \pm e\sin\theta}$ <u>Horizontal directrix</u>

10.8 Polar Eqns of Conics

Ex. 1 Determine the type of conic from the polar equation.

a) $r = \dfrac{15}{3-2\cos\theta}$

needs to be 1 $\rightarrow$ 3

$r = \dfrac{5}{1-\frac{2}{3}\cos\theta}$

$e = \frac{2}{3}$ $\quad e < 1$

Steps

1) Rewrite in the form of $r = \dfrac{d}{1 \pm e\cos\theta}$ (divide top/bottom by 3)

2) Use the value of "e" to determine which type of conic

The conic is an ellipse; has a vertical directrix, horizontal major axis $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

b) $r = \dfrac{6}{1-\cos\theta}$

$e = 1 \quad 1 = 1$

Parabola; vertical directrix; x = parabola

c) $r = \dfrac{4}{4-\cos\theta}$

has to become 1 $\rightarrow$ 4

$r = \dfrac{1}{1-\frac{1}{4}\cos\theta}$ $\quad e = \frac{1}{4}$ $\quad e < 1$

Ellipse with a vertical directrix; horizontal major axis

$\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

d) $r = \dfrac{4}{2+3\sin\theta}$

$r = \dfrac{2}{1+\frac{3}{2}\sin\theta}$ $\quad e = 3/2$ $\quad e > 1$ hyperbola with a horizontal directrix.

Equations of Conics: $\boxed{a}x^2 + \boxed{c}y^2 + dx + ey + f = 0$

Conic	Standard Form
Circle a = c	$(x-h)^2 + (y-k)^2 = r^2$
Parabola a*c = 0	$y = a(x-h)^2 + k$ **or** $x = a(y-k)^2 + h$
Ellipse a*c > 0	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ **or** $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
Hyperbola a*c < 0	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ **or** $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Polar Equations of Conics

Rectangular to polar: $\tan^{-1}\frac{y}{x} = \theta$ *and* $r^2 = x^2 + y^2$

Polar to rectangular: $x = r\cos\theta$ *and* $y = r\sin\theta$

A conic in polar form will look something like this: $r = \frac{d}{1 \pm e\cos\theta}$ **or** $r = \frac{d}{1 \pm e\sin\theta}$

accentricity

- Graph to see which shape the polar equation makes: Mode must be in polar and radians and use zoom fit

Ex. 1 Determine the type of conic represented by the equation

Equation	Type of Conic
1. $3x^2 - 2y^2 + 4y - 3 = 0$	a=3 Hyperbola c=-2 3×-2 $3\cdot(-2)=0$ $3\cdot(-2)>0$ $3\cdot -2<0$ ✓
2. $2y^2 - 3x + 2 = 0$	a=0 Parabola c=2 0×2 $0\cdot 2=0$ ✓
3. $x^2 + y^2 - 6x + 3y - 4 = 0$	a=1 c=1 1=1 ✓ Circle
4. $x^2 + 4y^2 - 2x - 3 = 0$	a=1 Ellipse c=4 1×4 $1\cdot 4=0$ $1\cdot 4>0$ ✓
5. $x^2 - 2x + 4y - 1 = 0$	a=1 Parabola c=0 1×0 $1\cdot 0=0$ ✓
6. $r = \frac{15}{3-2\cos\theta}$	Ellipse e is 2/3 ; e<1
7. $r = \frac{32}{3+5\sin\theta}$	Hyperbola e is 5/3 ; e>1
8. $r = \frac{6}{1-\cos\theta}$	Parabola e=1