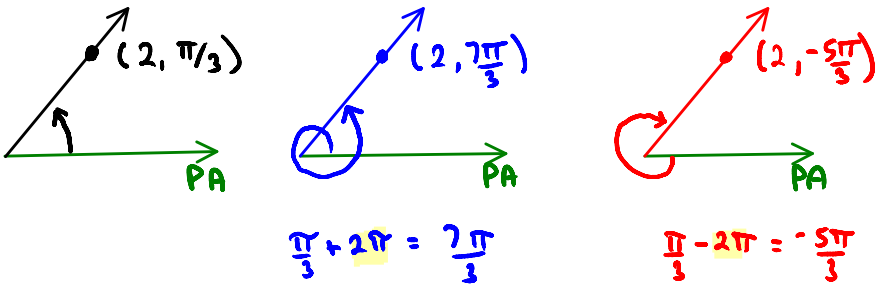
10.6 Polar Coordinates

Thursday, April 23, 2015
9:47 AM

* Polar Coordinates are not UNIQUE, unlike rectangular coordinates.



It is possible for r to be negative. The direction is OPPOSITE the terminal side of θ a distance of |r| from the pole (aka origin).

(r, θ) and $(-r, \theta + \pi)$ represent the same point.

$(r, \theta) = (r, \theta \pm 2\pi n)$ adding multiples of 2π (even)

$(r, \theta) = (-r, \theta \pm (2n+1)\pi)$ adding odd π

Ex. 2 Find 4 other polar coordinates of the given coordinate

a) $(3, \frac{2\pi}{3})$

$n=1$ $(r, \theta + 2\pi n) \rightarrow (3, \frac{2\pi}{3} + 2\pi(1)) \rightarrow (3, \frac{8\pi}{3})$
$n=-1$ $(r, \theta + 2\pi n) \rightarrow (3, \frac{2\pi}{3} + 2\pi(-1)) \rightarrow (3, -\frac{4\pi}{3})$

$n=0$ $(-r, \theta + (2n+1)\pi) \rightarrow (-3, \frac{2\pi}{3} + (2(0)+1)\pi) \rightarrow (-3, \frac{5\pi}{3})$
$n=1$ $(-r, \theta + (2n+1)\pi) \rightarrow (-3, \frac{2\pi}{3} + (2(1)+1)\pi) \rightarrow (-3, 11\pi/3)$

b) $(4, \frac{7\pi}{6})$

$(4, \frac{7\pi}{6} + 2\pi) \rightarrow (4, \frac{19\pi}{6})$
$(4, \frac{7\pi}{6} - 2\pi) \rightarrow (4, \frac{-5\pi}{6})$

* r stays the same
+2π or -2π

$(4, \frac{7\pi}{6} - 2\pi) \rightarrow (4, \frac{-5\pi}{6})$ } $+2\pi$ or -2π

$(-4, \frac{7\pi}{6} + \pi) \rightarrow (-4, \frac{13\pi}{6})$ } * r changes sign

$(-4, \frac{7\pi}{6} - \pi) \rightarrow (-4, \frac{\pi}{6})$ } $+\pi$ or $-\pi$

Ex.3 Determine if the given coordinates represent the given point. $(3, \frac{\pi}{6})$

a) $(3, \frac{13\pi}{6})$

$\frac{\pi}{6} + 2\pi = \frac{13\pi}{6}$ ✓

Yes

b) $(3, -\frac{5\pi}{6})$

$\frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$ X

No

c) $(-3, -\frac{5\pi}{6})$

$\frac{\pi}{6} - \pi = -\frac{5\pi}{6}$ ✓

Yes

Ex.4 Find 3 additional polar coordinates for

a) $(5, \frac{5\pi}{6})$ when $-2\pi \leq \theta \leq 2\pi$

* You can not add 2π, b/c it would be over 2π.

$\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$ $\quad \frac{5\pi}{6} + \pi = \frac{11\pi}{6}$ $\quad \frac{5\pi}{6} - \pi = -\frac{\pi}{6}$

$(5, -\frac{7\pi}{6})$ $\quad (-5, 11\pi/6)$ $\quad (-5, -\pi/6)$

b) $(-3, -\frac{3\pi}{4})$ when $-2\pi \leq \theta \leq 2\pi$

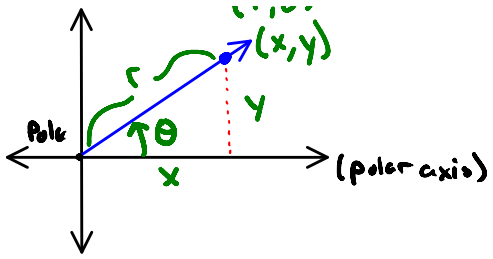
$-\frac{3}{4}\pi + 2\pi = \frac{5\pi}{4}$ $\quad -\frac{3\pi}{4} + \pi = \frac{\pi}{4}$ $\quad -\frac{3\pi}{4} - \pi = -7\pi/4$

$(-3, 5\pi/4)$ $\quad (3, \frac{\pi}{4})$ $\quad (3, -7\pi/4)$

Coordinate Conversion

(r, θ)

$\rightarrow (x, y)$

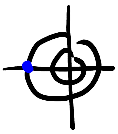


If P is a point with polar coordinate (r, θ), the rectangular coordinate (x,y) of P is:

$$x = r\cos\theta$$
$$y = r\sin\theta$$

Ex.5 Convert the polar points to rectangular coordinates

a) $(2, \pi)$



$x = 2\cos\pi \quad y = 2\sin\pi$

$x = 2(-1) \quad y = 2(0)$

$x = -2 \quad y = 0$

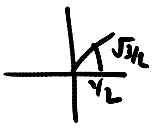
$(-2, 0)$

b) $(\sqrt{3}, \pi/6)$

$x = \sqrt{3}\cos\frac{\pi}{6} \quad y = \sqrt{3}\sin\frac{\pi}{6}$

$x = \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) \quad y = \sqrt{3}\left(\frac{1}{2}\right)$

$x = \frac{3}{2} \quad y = \frac{\sqrt{3}}{2}$



$(3/2, \sqrt{3}/2)$

Convert From Rectangular to Polar Coordinates

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2+y^2}$$

$$\tan\theta = \frac{y}{x} \quad \text{if } x \neq 0$$

Ex.6 Convert to Polar Coordinates

a) $(-1, 1)$

$r = \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}$

$r = \sqrt{2}$

$\tan\theta = \frac{1}{-1}$

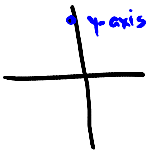
$\tan\theta = -1$

$\theta = -45 + 180$

$\theta = 135$ or $\frac{3\pi}{4}$

$(\sqrt{2}, 3\pi/4)$

b) $(0, 2)$



$r = \sqrt{(0)^2 + (2)^2}$

$r = \sqrt{4}$

$r = 2$

$\tan\theta = \frac{2}{0}$

$\theta = \frac{\pi}{2}$

$(2, \pi/2)$

Equation Conversion

Equation Conversion

To convert a rectangular equation to polar form:

1) replace x with $r\cos\theta$
2) replace y with $r\sin\theta$
3) solve for r

$y = x^2$ rectangular

$r\sin\theta = (r\cos\theta)^2$

$\frac{r\sin\theta}{r} = \frac{r^2\cos^2\theta}{r}$

$\sin\theta = r\cos^2\theta$

$\frac{\sin\theta}{\cos^2\theta} = \frac{r\cos^2\theta}{\cos^2\theta}$

tan: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Reciprocal: $\frac{1}{\cos\theta} = \sec\theta$

$r = \frac{\sin\theta}{\cos^2\theta} \rightarrow r = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$

$r = \tan\theta\sec\theta$ Simplest form

Ex.7 Convert from rectangular to Polar

a) $8xy = 12$

$8(r\cos\theta)(r\sin\theta) = 12$

$8r^2\cos\theta\sin\theta = 12$

$4r^2(2\cos\theta\sin\theta) = 12$

$4r^2\sin 2\theta = 12$

$r^2\sin 2\theta = 3$

FYI

$\sin 2\theta = 2\cos\theta\sin\theta$

Double $\measuredangle$

Calculator

- mode

Func Para Pol Seq

Instead of x, θ

b) $y = -5$

$\frac{r\sin\theta}{\sin\theta} = \frac{-5}{\sin\theta}$

$r = \frac{-5}{\sin\theta}$ $\quad \frac{1}{\sin\theta} = \csc\theta$

$r = -5\csc\theta$

c) $3x - 2y = 1$

$3r\cos\theta - 2r\sin\theta = 1$

$r(3\cos\theta - 2\sin\theta) = 1$

$r = \frac{1}{3\cos\theta - 2\sin\theta}$

Ex.8 Transform from polar eqn to rectangular.

a) $r = 6\cos\theta$

a) $r = 6\cos\theta$

$r(r) = r(6\cos\theta)$

$r^2 = 6r\cos\theta$

$x^2+y^2 = 6x$

$x^2-6x+y^2 = 0$

$(x^2-6x+\underline{9})+y^2 = \underline{+9}$

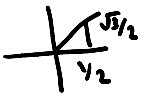
$(x-3)^2+y^2 = 9$

$(3,0)$ center

$3 = r$

1) multiply both sides by r
2) $x = r\cos\theta$
3) $r^2 = x^2+y^2$
4) Complete the square for x
5) Eqn of a Circle $r^2 = x^2+y^2$

b) $\theta = \frac{\pi}{3}$ $\tan\theta = \frac{y}{x}$ $\tan\theta = \frac{\sqrt{3}/2}{1/2}$



$y = \sqrt{3}x$

$\tan\theta = \sqrt{3}$

$\theta = y$

$x = \frac{\pi}{3}$

c) $r = 4\sin\theta$

$r^2 = 4r\sin\theta$

$x^2+y^2 = 4y$

$x^2+y^2-4y = 0$

$x^2+(y^2-4y+\underline{4}) = \underline{4}$

$x^2+(y-2)^2 = 4$ Circle

d) $r(-2\sin\theta+3\cos\theta) = 2$

$-2r\sin\theta+3r\cos\theta = 2$

$-2y+3x = 2$

$y = \frac{3}{2}x-1$ Linear

e) $\theta+\frac{\pi}{4} = 0$

$\theta = -\frac{\pi}{4}$

$\tan\theta = \tan\frac{-\pi}{4}$

tan of both sides

$\tan\theta = \sin\theta \rightarrow r\sin\theta = y$

$\tan\theta = \tan\frac{-\pi}{4}$

$\frac{y}{x} = -1$

$y = -x$

$\tan\theta = \frac{\sin\theta}{\cos\theta} \rightarrow \frac{r\sin\theta}{r\cos\theta} = \frac{y}{x}$

Hw To pg 743 #'s 1,3,5,7,12,17,22,31,33,36,45a,49a, 51,61,57