10.5 Parametric Equations Name: ______________________

Up until this point we have looked at functions that have two variables, which are typically x and y. We are now going to introduce a third variable called a parameter (t)

We will define both x and y in terms of the parameter, and as the parameter changes our x and y values will change.

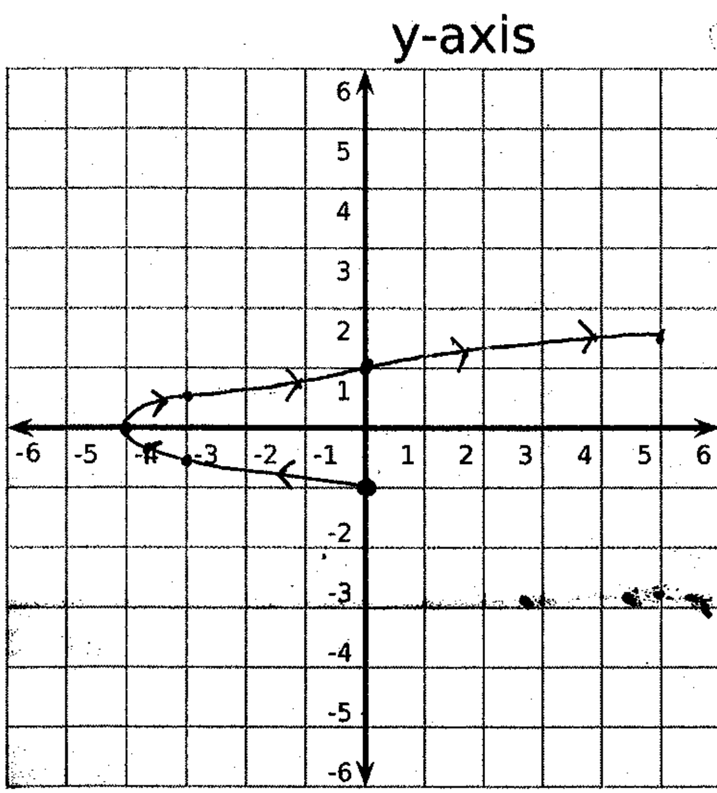
Example 1: Sketching plane ~~cure~~ curve $x = t^2 - 4$ *and* $y = \frac{t}{2}$, $\boxed{-2 \le t \le 3}$ ← This drives the bus!

- First make a table to determine the set of coordinates (x,y) from a chosen t value.

t	-2	-1	0	1	2	3
x	$(-2)^2-4$ 0	$(-1)^2-4$ -3	-4	-3	0	5
y	$\frac{(-2)}{2}$ -1	$\frac{(-1)}{2}$ $-1/2$	0	$1/2$	1	1.5

* Use the eqns given, plug "t" into them! *

- Plot the points in order of increasing values of t to determine the **orientation** of the curve. Use arrows to indicate the direction.



If this was a particle moving along this curve, it would start at $t=-2$ $(0,-1)$ and follow the curve till $t=3$ $(5, 3/2)$.

This parametric eqn. is not a function; it fails the VLT!

Example 2: Graphing in Calculator: Change your mode to be in parametric.

Use a graphing utility to graph the curves. Which curve is y a function of x? (function = passes vertical line test)

Change your window settings: $-6 \le t \le 10$

a. $x = t^2$ *and* $y = t^3$

Not a function

b. $x = t$ *and* $y = t^3$

Function

c. $x = t^2$ *and* $y = t$

Not a function

d) $x = \frac{1}{2}t^3$
$y = 6t$
Is a function

e) $x = 3t^2 + 4$
$y = t - 3$
Not a function

Converting from parametric to rectangular: Eliminate the parameter

Steps:	1. Given parametric equation	2. solve for t in one equation *Choose Simplest Eqn! *	3. substitute in second equation	4. solve for rectangular equation
Example:	$x = t^2 - 4$ $y = \frac{1}{2}t$	$y = \frac{1}{2}t \rightarrow 2y = t$	$x = t^2 - 4$ $x = (2y)^2 - 4$	$x = 4y^2 - 4$ Parabola, opens Right
	$x = t+1$ $y = t^2$	$x = t+1$ $t = x - 1$	$y = t^2$ $y = (x-1)^2$	$y = (x-1)^2$ or $y = x^2 - 2x + 1$ Parabola, opens up

Sometimes the domain and range cause problems, and you have to restrict them.

1. Find the domain and range of x and y when it is written as a parametric equation
2. Convert the parametric equation to rectangular.
3. Match up the domain and range for the rectangular equation.

Example 3: Identify the curve represented by the equations: $x = \frac{1}{\sqrt{t+1}}$ $y = \frac{t}{t+1}$

① $x = \frac{1}{\sqrt{t+1}}$

$t+1 > 0$

$\boxed{t > -1}$

$y = \frac{t}{t+1}$

$t+1 \neq 0$

$\boxed{t \neq -1}$

② $x = \frac{1}{\sqrt{t+1}}$

$(t+1)x^2 = \frac{1}{\cancel{t+1}}\cancel{(t+1)}$

$\frac{(t+1)x^2}{x^2} = \frac{1}{x^2}$

$t+1 = \frac{1}{x^2}$

$t = \frac{1}{x^2} - 1$

$y = \dfrac{\frac{1}{x^2} - 1}{\frac{1}{x^2} - 1 + 1}$

$y = \dfrac{\frac{1}{x^2} - 1}{\frac{1}{x^2}}$

KCF

$\left(\frac{1}{x^2} - 1\right)\left(\frac{x^2}{1}\right)$

$y = \frac{x^2}{x^2} - x^2$

$y = 1 - x^2$

$y = -x^2 + 1$ Parabola

③ restrict the domain of the converted eqn so that x is all positive. $x = 0, x = 1, x = 2, \ldots$

Example 4: Identify the cure represented by $x = 3\cos\theta$ $y = 4\sin\theta$, $0 \leq \theta \leq 2\pi$ (Theta is your parameter instead of t) * Solve for cos and sin

$x = 3\cos\theta$ $y = 4\sin\theta$

$\cos\theta = \frac{x}{3}$ $\sin\theta = \frac{y}{4}$

* Use Pythag. ID

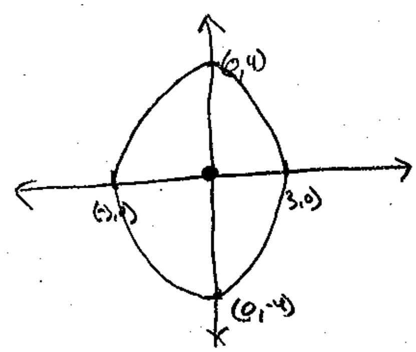
$\sin^2\theta + \cos^2\theta = 1$; this will form an eqn involving x and y!

$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$

$\frac{x^2}{9} + \frac{y^2}{16} = 1$

Ellipse



. An elliptical curve is traced CCW as θ varies from 0 to 2π.

Example 5: Finding parametric equations for a given graph:

Find a set of parametric equations to represent the graph of $y = 1 - x^2$ using the following parameters.

a. t = x

$y = 1 - (t)^2$

$y = 1 - t^2$

$x = t$
$y = 1 - t^2$

b. t = 1 − x Solve for x

$x = 1 - t$
$y = -t^2 + 2t$

$y = 1 - (1-t)^2$

$y = -t^2 + 2t$