10.3 Hyperbolas

Hyperbola: the set of all points in a plane such that the absolute value of the difference of the distances from two fixed points called the foci, is constant.

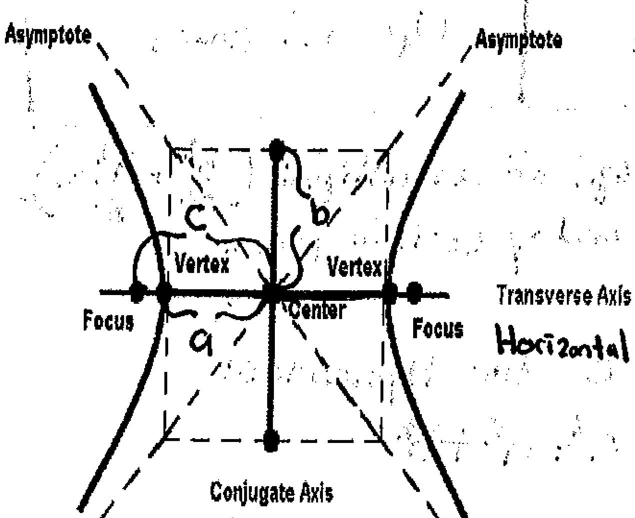
Center (h,k): the midpoint of the segment connecting the foci (or vertices of a hyperbola.

Vertex: a point on each branch of the hyperbola that is nearest the center. 2 vertices

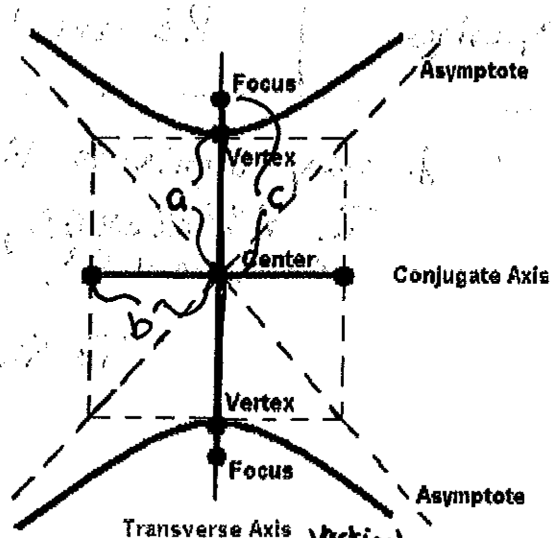
Asymptote: as a hyperbola recedes from the center, the branches approach these lines. Asymptotes intersect at the center!

Transverse Axis: segment of length **2a** whose endpoints are vertices of the hyperbola (a is the value under the positive term).

Conjugate Axis: segment of the length **2b** that is perpendicular to the transversal axis at the center (b is the value under the negative term)



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

10.3 Hyperbolas

Equations of Hyperbolas: Center (h,k)

Standard form of equation (a is under the positive term)	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
direction of transverse axis (determined by which term is positive)	Horizontal	Vertical
foci x2	$(h \pm c, k)$	$(h, k \pm c)$
vertices x2	$(h \pm a, k)$	$(h, k \pm a)$
length of transverse axis (major)	2a (units)	2a (Units)
length of conjugate axis (minor)	2b (units)	2b (units)
equation of asymptotes Slope = $\frac{y}{x}$ $y = mx + b$	$y = k \pm \frac{b}{a}(x-h)$ $m = \frac{b}{a}$	$y = k \pm \frac{a}{b}(x-h)$ $m = \frac{a}{b}$
Direction of Openings	Rt and Lft	Up and down

* Very similar to the eqn of an ellipse $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right)$. In HYPERBOLAS, x and y switch places, not a^2 and b^2.

To find "c" for Hyperbolas:

$c^2 = a^2 + b^2$

10.3 Hyperbolas

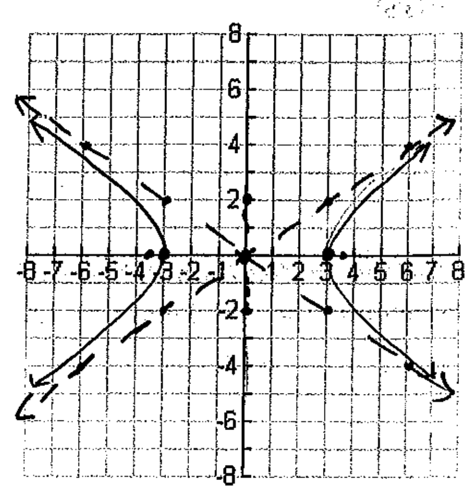
To draw the graph: 1) draw the asymptotes, 2) the point of intersection of the diagonals is the center of the hyperbola

1. Graph $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$a^2 = 9$ $b^2 = 4$ $c^2 = 9 + 4$

$a = 3$ $b = 2$ $c^2 = 13$

$c \approx 3.61$



Center: $(0,0)$

Vertices: $(-3,0)$ $(3,0)$

Vertices $(h \pm a, k)$

$(0+3,0)$ $(0-3,0)$

$(3,0)$ $(-3,0)$

Foci: $(-3.61,0)$ $(3.61,0)$

Foci $(h \pm c, k)$

$(0+3.61, 0)$ $(0-3.61,0)$

$(3.61,0)$ $(-3.61,0)$

Asymptotes: $y = \pm \frac{2}{3}x$

Asymptotes $y = k \pm \frac{b}{a}(x-h)$ $y = 0 \pm \frac{2}{3}(x-0)$

Length and direction of transverse axis:

Horizontal $2a$

$2(3) = 6$ units

Length and direction of conjugate axis:

Vertical $2b$

$2(2) = 4$ units

2. Write an equation of a hyperbola with foci at (0,7) and (0,-7) if the length of the transverse axis is 6 units.

Graph

SAME

Vertical TA

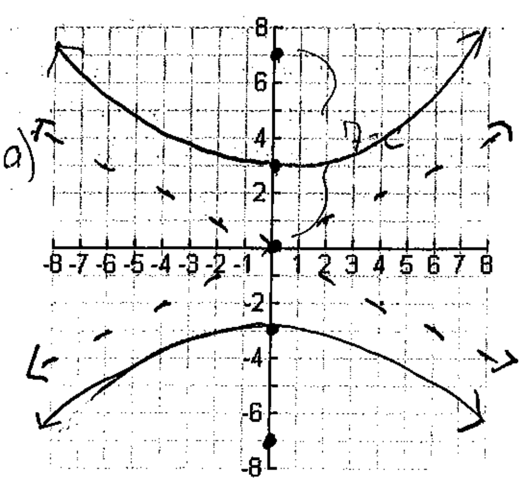
$TA = 6$

$6 = 2a$

$a = 3$

Center

$(0,0)$



$c = 7$

$c^2 = 49$

$49 = 9 + b^2$

$b^2 = 40$

$b \approx 6.32$

$$\frac{y^2}{9} - \frac{x^2}{40} = 1$$

$CA = 2b$

$2(6.32)$

12.64

Vertices $(h, k \pm a)$

$(0,3)$ $(0,-3)$

Asymptotes

$y = k \pm \frac{a}{b}(x-h)$

$y = 0 \pm \frac{3}{6.32}(x-0)$

$y \approx \pm \frac{1}{2}x$

3. Determine the standard form of the equation, and determine the coordinates of the vertices and foci.

$4x^2 - 9y^2 - 32x - 18y + 19 = 0$

$4x^2 - 32x - 9y^2 - 18y = -19$

$4(x^2 - 8x + \_\_) - 9(y^2 + 2y + \_\_) = -19 + (4 \cdot \_\_) + (-9 \cdot \_\_)$

$4(x^2 - 8x + 16) - 9(y^2 + 2y + 1) = -19 + (4 \cdot 16) + (-9 \cdot 1)$

$$\frac{4(x-4)^2}{36} - \frac{9(y+1)^2}{36} = \frac{36}{36}$$

$$\frac{(x-4)^2}{9} - \frac{(y+1)^2}{4} = 1$$

Center

$(4,-1)$

TA is Horizontal

$a = 3$ $b = 2$

$c = \sqrt{13} \approx 3.61$

Foci

$(7.61, -1)$

$(.39, -1)$

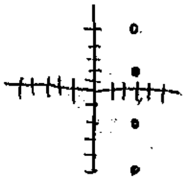
Vertices

$(7, -1)$

$(1, -1)$

10.3 Hyperbolas

4. Find the standard form of the equation of the hyperbola with vertices at (3,2) and (3,-2) and foci at (3,-5) and (3,5).



$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1 \qquad \frac{y^2}{4}-\frac{(x-3)^2}{21}=1$$

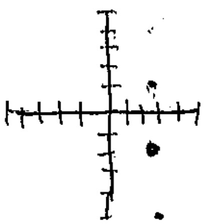
Center
(3,0)

$a=2$
$a^2=4$

$c=5$
$c^2=25$

$25=4+b^2$
$b^2=21$

5. Find the standard form of the equations of the hyperbola having vertices at (3,-5) and (3,1) and with the asymptotes: $y = 2x - 8$ and $y = -2x + 4$



$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1 \qquad \frac{(y+2)^2}{9}-\frac{(x-3)^2}{9/4}=1$$

Use the midpoint to find the center

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

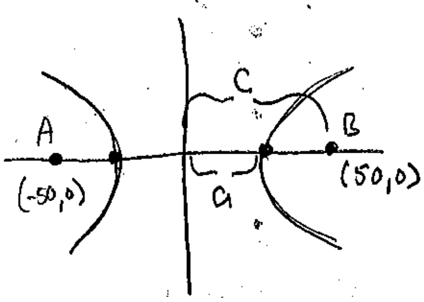
Center
(3,-2)

$m=\frac{a}{b}$
$m=2$

$a=3$
$a^2=9$

$2=\frac{3}{b}$ $\quad b=3/2$ $\quad b^2=\frac{9}{4}$

6. The LORAN navigational system is based on hyperbolas. Two stations send out signals at the same time. A ship notes the difference in the times at which it receives the signals. The ship is on a hyperbola with the stations at the foci. Suppose a ship determines that the difference of its distance from two stations is 50 nautical miles. The stations are 100 nautical miles apart. Write an equation for a hyperbola on which the ship lies if the stations are (-50,0) and (50,0).



$c=50$
$c^2=2500$

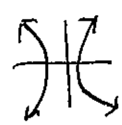
Transverse Axis
$2a=50$
$a=25$
$a^2=625$

$c^2=a^2+b^2$
$(50)^2=(25)^2+b^2$
$2500=625+b^2$
$1875=b^2$

$$\frac{x^2}{625}-\frac{y^2}{1875}=1$$

- Eccentricity of a hyperbola $\frac{c}{a}$: (because $c > a$ it follows that $e > 1$) If the eccentricity is large the branches are nearly flat, and if it is close to 1 they are more pointed.

"e" close 1



"e" is larger (flat)

1. Find the Eccentricity of the hyperbola. $\frac{(y+2)^2}{9}-\frac{(x-3)^2}{64}=1$

$e=\frac{c}{a}$

$c^2=a^2+b^2$
$c^2=9+64$
$c^2=73$
$c\approx 8.54$

$e=\frac{8.54}{3}$

$e\approx 2.85$