10.2 Circles and Ellipses

Ex. 5 Determine the coordinates of the foci and the length of the major and minor axes of an ellipse whose equation is given and then graph.

a) $\frac{49x^2}{784} + \frac{16y^2}{784} = \frac{784}{784}$

$\frac{x^2}{16} + \frac{y^2}{49} = 1$

Center (h,k)
$(0,0)$

vertical

maj: $a^2 = 49$, $a = 7$, $2a$: 14 units

min: $b^2 = 16$, $b = 4$, $2b$: 8 units

Foci $(h, k \pm c)$

$(0, 0+5.74) \rightarrow (0, 5.74)$

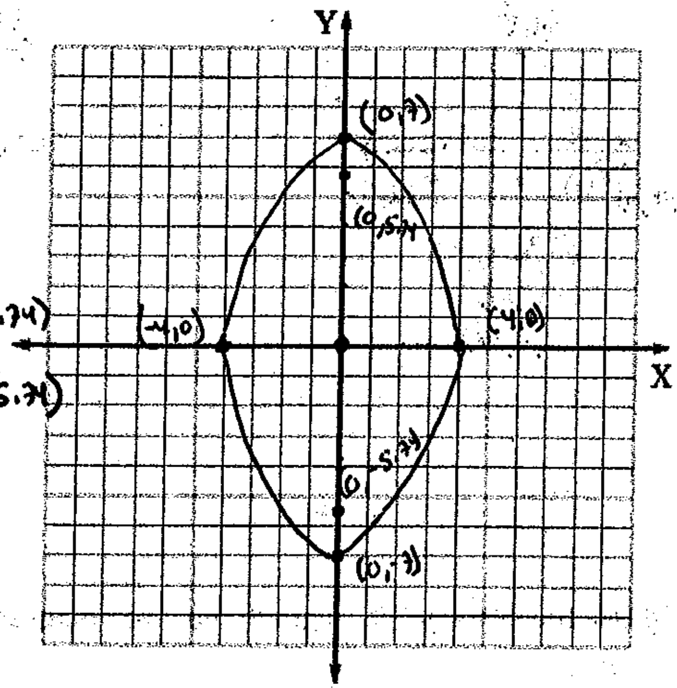
$(0, 0-5.74) \rightarrow (0, -5.74)$

$c^2 = a^2 - b^2$

$c^2 = 49 - 16$

$c^2 = 33$

$c \approx 5.74$



b) $\frac{9x^2}{144} + \frac{16y^2}{144} = \frac{144}{144}$

$\frac{x^2}{16} + \frac{y^2}{9} = 1$

Center (h,k)
$(0,0)$

Horizontal

maj: $a^2 = 16$, $a = 4$, $2a$: 8 units

min: $b^2 = 9$, $b = 3$, $2b$: 6 units

$c^2 = a^2 - b^2$

$c^2 = 16 - 9$

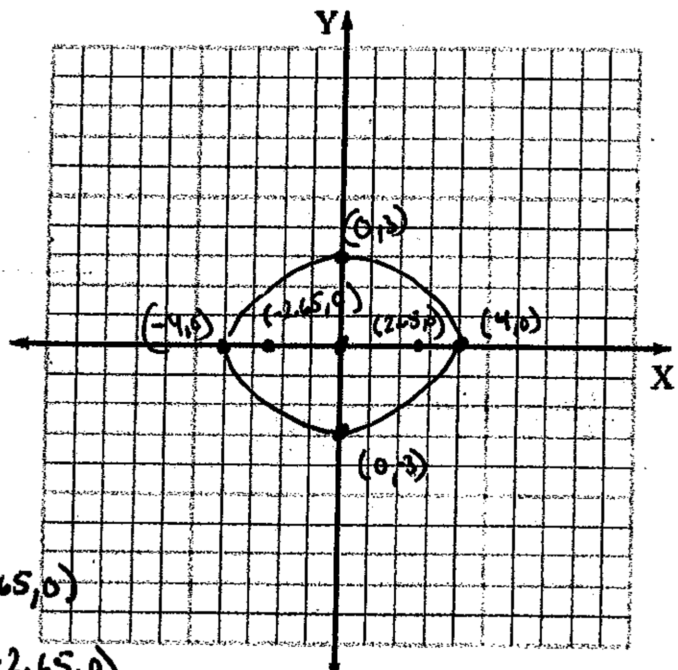
$c^2 = 7$

$c \approx 2.65$

Foci $(h \pm c, k)$

$(0+2.65, 0) \rightarrow (2.65, 0)$

$(0-2.65, 0) \rightarrow (-2.65, 0)$



10.2 Circles and Ellipses

c) $x^2 + 4y^2 + 4x - 24y + 24 = 0$

* Complete the square twice *

$(x^2+4x+\_\_)+(4y^2-24y+\_\_)=-24$

$(x^2+4x+\_\_)+4(y^2-6y+\_\_)=-24+\_\_+(4\cdot\_\_)$

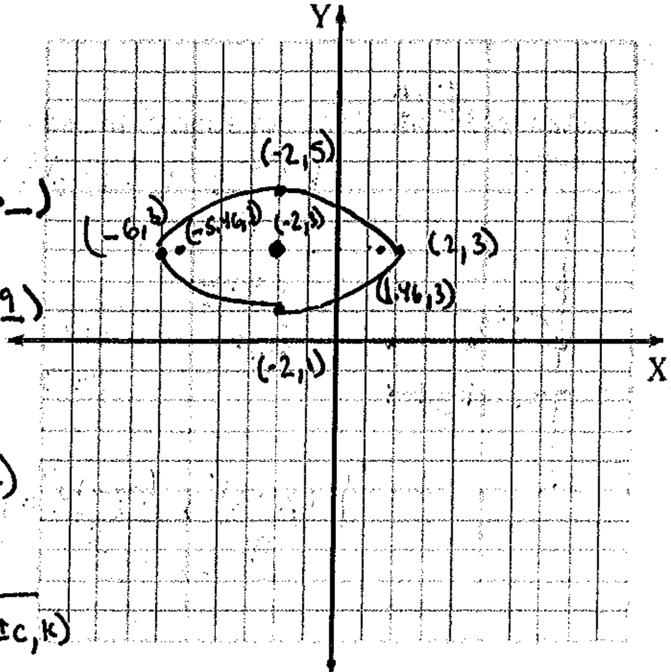
$\left(\frac{4}{2}\right)^2=4$ $\quad\left(\frac{-6}{2}\right)^2=9$

$(x^2+4x+\underline{4})+4(y^2-6y+\underline{9})=-24+\underline{4}+(4\cdot\underline{9})$

$\frac{(x+2)^2}{16}+\frac{4(y-3)^2}{16}=\frac{16}{16}$

$\frac{(x+2)^2}{16}+\frac{(y-3)^2}{4}=1$

center (h,k)
$(-2,3)$



a	b	c
$a^2=16$	$b^2=4$	$c^2=16-4$
$a=4$	$b=2$	$c^2=12$
maj	min	$c\approx3.46$
8 units	4 units	

Foci $(h\pm c,k)$

$(-2+3.46,3)$ $\quad(-2-3.46,3)$

$(1.46,3)$ $\quad(-5.46,3)$

d) $16x^2 + 9y^2 - 96x - 90y + 225 = 0.$

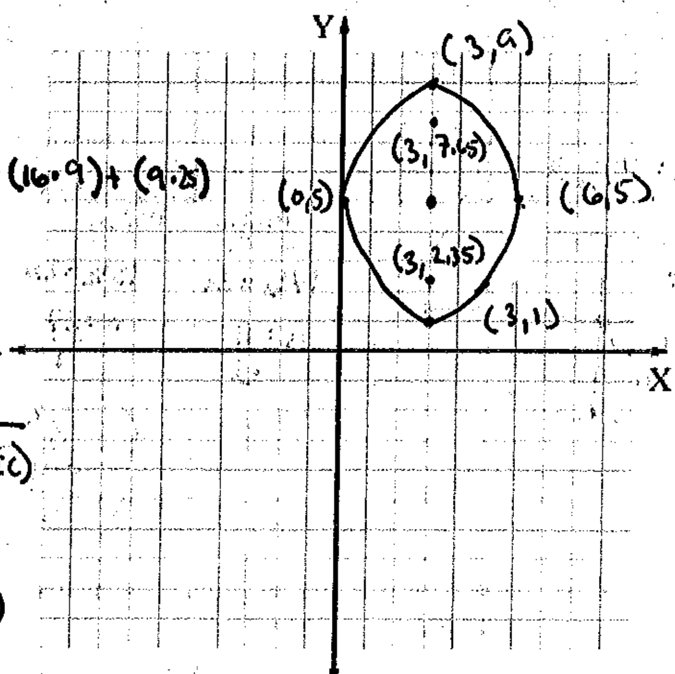
$(16x^2-96x+\_\_)+(9y^2-90y+\_\_)=-225$

$16(x^2-6x\,\underline{9})+9(y^2-10y+\underline{25})=-225+(16\cdot9)+(9\cdot25)$

$\frac{16(x-3)^2}{144}+\frac{9(y-5)^2}{144}=\frac{144}{144}$

$\frac{(x-3)^2}{9}+\frac{(y-5)^2}{16}=1$

Center
$(3,5)$



a	b	c
$a^2=16$	$b^2=9$	$c^2=16-9$
$a=4$	$b=3$	$c^2=7$
		$c\approx2.65$

Foci $(h,k\pm c)$

$(3,7.65)$

$(3,2.35)$

In the above example, describe what the coefficients of x^2 and y^2 in the standard equation tell about the ellipse.

10.2 Circles and Ellipses

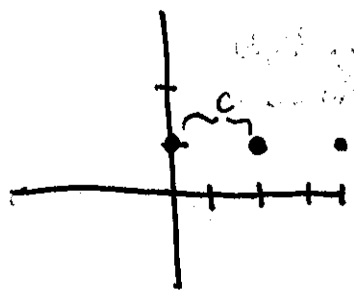
Ex. 6

y-values same horizontal

Find the standard ~~from~~ form of the equation of the ellipse having foci at (0,1) and (4,1) and a major axis of length 6.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

Foci are located on major axis.



Center: (2,1)

c: $c=2$, $c^2=4$

Major axis: 6 units, $2a=6$, $a=3$, $a^2=9$

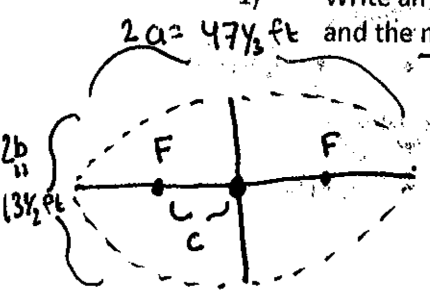
$c^2=a^2-b^2$, $4=9-b^2$, $b^2=5$

$$\frac{(x-2)^2}{9}+\frac{(y-1)^2}{5}=1$$

EX. 7 Applications

a) In an ellipse, sound or light coming from one focus is reflected to the other focus. In a whispering gallery, a person can hear another person whisper from across the room if the two people are standing at the foci. The whispering gallery at the Museum of Science and Industry in Chicago has an elliptical cross section that is 13 feet 6 inches by 47 feet 4 inches.

1) Write an equation to model this ellipse. Assume that the center is at the origin and the major axis is horizontal.



Major: $47\frac{1}{3}=2a$, $a=\frac{71}{3}$

Minor: $13\frac{1}{2}=2b$, $b=\frac{27}{4}$

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$\frac{x^2}{(71/3)^2}+\frac{y^2}{(27/4)^2}=1$$

$$\frac{x^2}{\frac{5041}{9}}+\frac{y^2}{\frac{729}{16}}=1$$

2) How far apart are the points at which two people should stand to hear each other whisper?

2c

$c^2=\frac{5041}{9}-\frac{729}{16}$

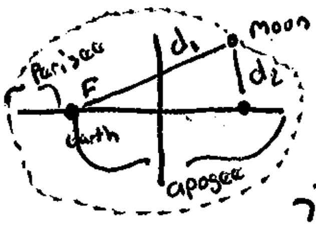
$c^2=\frac{74{,}095}{144}$

$c \approx 22.68$

$2(22.68)=45.36$ ft apart

10.2 Circles and Ellipses

b) The moon travels about the earth in an elliptical orbit with the earth at one focus. The major and minor axes of the orbit have lengths of 768, 806 km and 767,746 km, respectively. Find the greatest and least distances (the apogee and perigee) from the earth's center to the moons center.



* b/c the earth is a "focus" and the moon is a point on the ellipse, the greatest distance between the center of the earth/moon is: "a+c"

MAJ
768,806
$2a = 768,806$
$a = 384,403$

Min
767,746
$2b = 767,746$
$b = 383,873$

$c^2 = (384,403)^2 - (383,873)^2$

$c^2 = 407,186,280 \quad c \approx 20,178.36$

Apogee (A+c)
404,581.86 Km

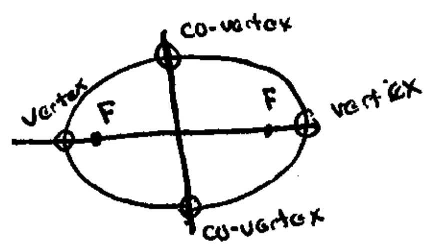
Perigee (A−c)
364,224.14 Km

Ellipses have practical and aesthetic uses: machine gears, supporting arches, acoustical designs, and the orbits of satellites and planets. One reason it was difficult for early astronomers to detect that the orbits of the planets are ellipses is that the foci of the planetary orbits are relatively close to their centers, and so the orbits are nearly circular.

To measure the ***ovalness*** of an ellipse, you can use **ECCENTRICITY** given by the ratio $e = \frac{c}{a}$ where ***0 < e < 1*** for every ellipse. An ellipse that is close to circular will have a small eccentricity ratio, but an elongated ellipse has eccentricity ratio closer to 1.

- An ellipse that is more circular has foci closer to the center.

- An ellipse that is more elongated has foci closer to the vertices.



<u>**Ex. 8**</u> Find eccentricity of the given ellipse.

a) $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{9} = 1$

$a^2 = 25 \quad b^2 = 9$

$a = 5 \quad b = 3$

$c^2 = 25 - 9$

$c^2 = 16$

$c = 4$

eccentricity

$e = \frac{c}{a}$

$e = \frac{4}{5}$

$e = .8$

closer to 1, so more elongated!