10.2 Circles and Ellipses

Circle

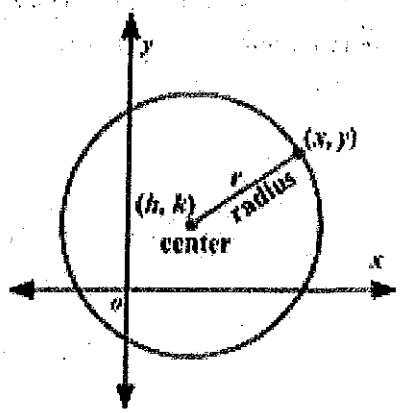
Is a set of all points in a plane that is EQUIDISTANT from a fixed center. A line segment that has an endpoint at the center and on the circle is called the Radius.

Equation of a circle

$(x-h)^2+(y-k)^2=r^2$

center **(h, k)**
radius **r**

notice that r is not squared!



Ex 1. Write the equation for each circle.

a) center (-6,9) and r=15

$(x-h)^2+(y-k)^2=r^2$

$(x+6)^2+(y-9)^2=225$

b) diameter with endpoints (2, 8) and (-4, -2)

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\quad mp=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$mp=\left(\frac{2+(-4)}{2},\frac{8+(-2)}{2}\right)$

aka center $(-1,3)$

center	EP
(-1,3)	(2,8)

distance $\rightarrow d=\sqrt{(-1-2)^2+(3-8)^2}$

aka $r=\sqrt{34}\rightarrow r^2=34$

$(x+1)^2+(y-3)^2=34$

Ex. 2 Determine the center and radius of the circle with the given equations.

a) $x^2+y^2=10$

$(x-h)^2+(y-k)^2=r^2$

center $(0,0)$

$r=\sqrt{10}$

b) $x^2+y^2+2x-12y-12=0$

$(x^2+2x+\_\_)+(y^2-12y+\_\_)=12+\_+\_$

$(\frac{2}{2})^2\rightarrow 1$ $\quad (\frac{-12}{2})^2\rightarrow 36$

$(x^2+2x+1)+(y^2-12y+36)=12+1+36$

$(x+1)^2+(y-6)^2=49$

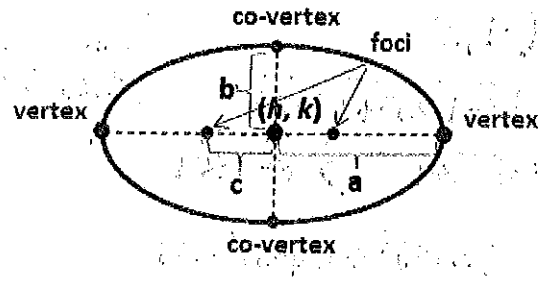
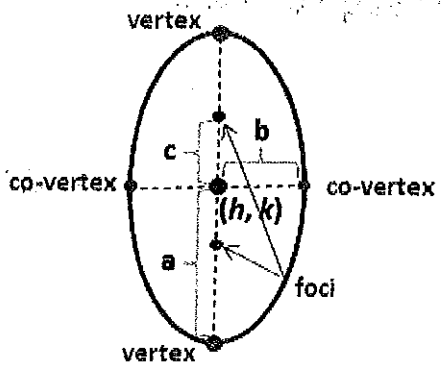
center $(-1,6)$ $\quad r=7$

10.2 Circles and Ellipses

Ellipse

Is the set of all points in a plane such that the sum of the Distances from two fixed points (called Foci) is constant.

Every ellipse has two Axes of Symmetry. The points at which the ellipse Intersects its axes of symmetry determine two segments with endpoints on the ellipse.

Horizontal Ellipse	Vertical Ellipse
At (0, 0): $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	At (0, 0): $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
General: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	**General:** $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$
$a^2-b^2=c^2$	$a^2-b^2=c^2$
Center: (h,k) **Foci:** $(h\pm c,k)$	**Center:** (h,k) **Foci:** $(h,k\pm c)$
Vertices: $(h\pm a,k)$ **Co-Vertices:** $(h,k\pm b)$	**Vertices:** $(h,k\pm a)$ **Co-Vertices:** $(h\pm b,k)$
	

The Major axis is the longer segment (AOS). The length of the major axis is **2a**.

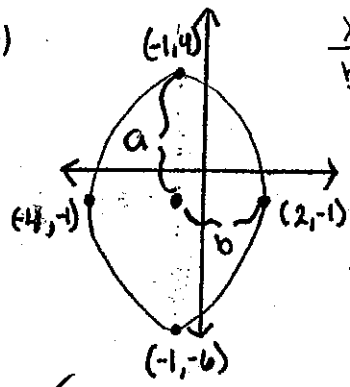
The Minor axis is the shorter segment (AOS). The length of the minor axis is **2b**.

The Foci are always on the MAJOR AXIS. The Center (h, k) is the intersection of the two axes.

10.2 Circles and Ellipses

Ex. 3 Write the equation of an ellipse from the given ellipses, find the foci, and the length of the major and minor axes.

a)



$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$

Center
(-1,-1)

a=5 b=3

$c^2 = a^2 - b^2$

$c^2 = 25 - 9$

$c = 4$

Foci $(h, k \pm c)$

$(-1, -1+4)$ $(-1, -1-4)$

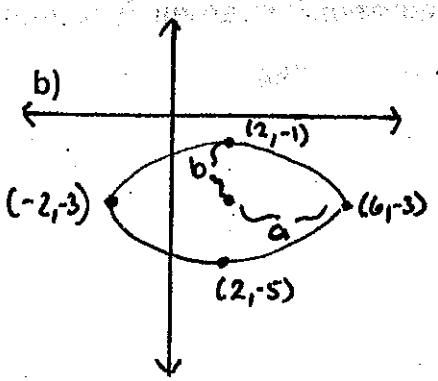
$(-1, 3)$ $(-1, -5)$ ✓

$\frac{(x+1)^2}{9} + \frac{(y+1)^2}{25} = 1$ ✓

Length

✓ maj $2a \to 2(5) = 10$ min $2b \to 2(3) = 6$

b)



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Center
(2,-3)

a=4 b=2

c

$c^2 = 16 - 4$

$c^2 = 12$

$c = \sqrt{12} \to 2\sqrt{3}$ or 3.46

Foci $(h \pm c, k)$

$(2+3.46, -3)$ $(2-3.46, -3)$

$(5.46, -3)$ $(-1.46, -3)$

$\frac{(x-2)^2}{16} + \frac{(y+3)^2}{4} = 1$

Length

maj $2a$ = 8 min $2b$ = 4

Ex. 4 Use the same graph to graph the given ellipses.

a) $\frac{(x+1)^2}{4} + \frac{(y+2)^2}{9} = 1$ Blue Vertical ellipse

center $(-1, -2)$

$a = 3$ $b = 2$ $c^2 = a^2 - b^2$

$c^2 = 9 - 4$

$c^2 = \sqrt{5} \approx 2.24$

Foci $(h, k \pm c)$

$(-1, -2+2.24)$ $(-1, -2-2.24)$

$(-1, .24)$ $(-1, -4.24)$

b) $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ Green Horizontal

center $(1, 2)$

$a = 3$ $b = 2$ $c = \sqrt{5} \approx 2.24$

Foci $(h \pm c, k)$

$(1+2.24, 2)$ $(1-2.24, 2)$

$(3.24, 2)$ $(-1.24, 2)$