10.1 Intro to Conics: **Parabola**

Conic Section (aka Conics)

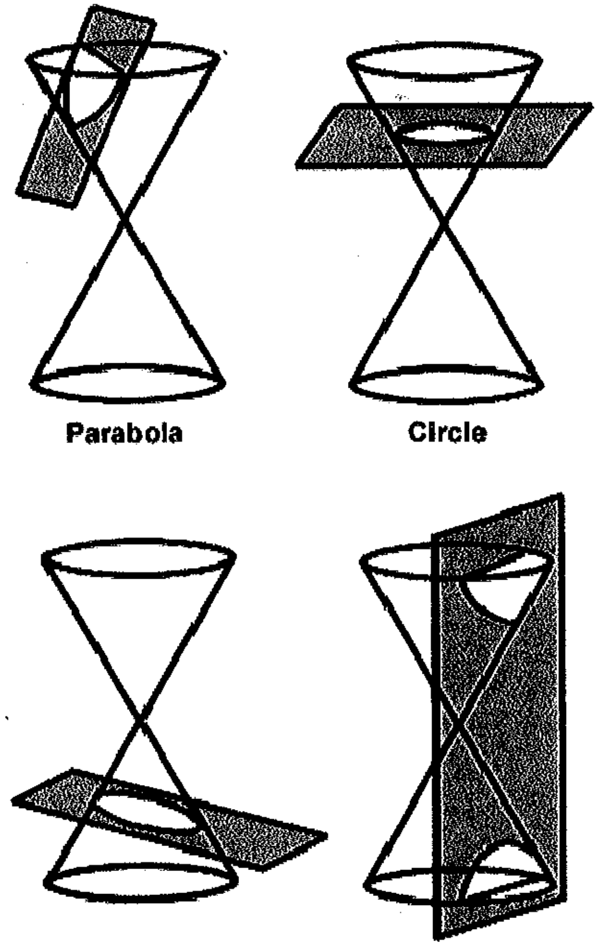
Is the INTERSECTION of a plane and a double-napped cone.

Conics defined algebraically are in general a 2nd degree equation.

ie $y = x^2 + x + 2$ $\quad x = -\frac{1}{2}y^2$

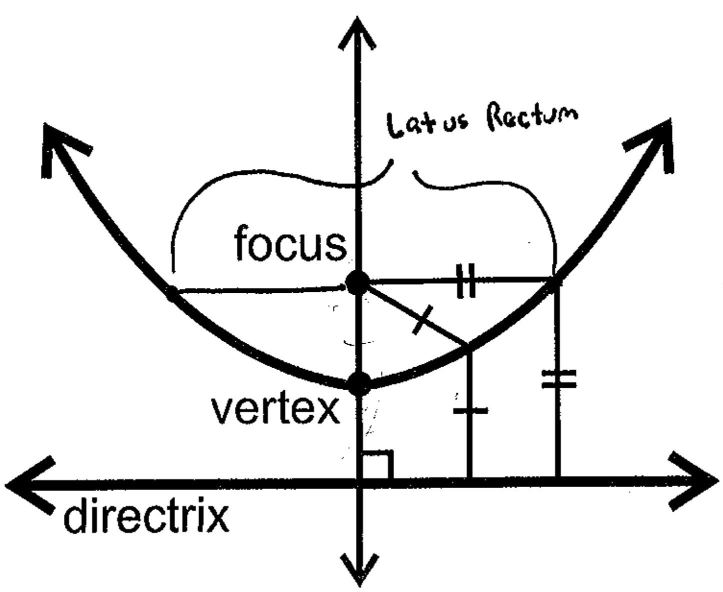
$x^2 + y^2 = 9$

Conics defined geometrically are a collection of points or LOCUS of points satisfying a geometric property.



Parabola

Is the set of all points (x, y) that are EQUIDISTANT from a fixed line called the DIRECTRIX and a fixed point called the FOCUS not on the directrix.



The midpoint between the focus and the directrix is called the VERTEX.

LATUS RECTUM is the line segment that goes through the focus, is perpendicular to the major axis, and has both end points on the parabola.

*(The length of the Latus Rectum is calculated different for each conic.)

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There are two types of a parabola:

	$y = x^2$	$x = y^2$
Vertex Form of Eqn	$y = a(x-h)^2 + k$	$x = a(y-k)^2 + h$
Vertex	(h, k)	(h, k)
AOS	$x = h$ vertical dashed line	$y = k$ horizontal dashed line
Focus	$\left(h, k + \frac{1}{4a}\right)$ A point (x, y)	$\left(h + \frac{1}{4a}, k\right)$ A point (x, y)
Directrix	$y = k - \frac{1}{4a}$ horizontal dashed line	$x = h - \frac{1}{4a}$ vertical dashed line
Direction of Opening	$a > 0$ up $a < 0$ down	$a > 0$ right $a < 0$ left
Latus Rectum	$\left\lvert \frac{1}{a} \right\rvert$	$\left\lvert \frac{1}{a} \right\rvert$
LR Endpoints	$\left\lvert \frac{1}{2a} \right\rvert$ add and subtract from h 2 points (endpoints of the LR)	$\left\lvert \frac{1}{2a} \right\rvert$ add and subtract from k

"Completing the Square" Review:

1) Move the constant to the other side
2) Make the coefficient of x^2 one (divide EVERYTHING)
3) $\left(\frac{b}{2}\right)^2$ is added to both sides of the equation
4) Factor the "perfect square" trinomial
5) Solve for either y or x

$y = x^2 + 4x + 8$

$y - 8 = x^2 + 4x$

$\left(\frac{4}{2}\right)^2 = 4$

$y - 8 + 4 = x^2 + 4x + 4$

$y - 4 = (x+2)^2$

$y = (x+2)^2 + 4$

(vertex Form)

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Ex. 1 Write the equation of a parabola in vertex form, identify the vertex, AOS, direction of opening, focus, directrix, latus rectum, the end points of the LR, and graph it.

a) $y = 2x^2 + 12x + 10$

$y = (2x^2 + 12x + \_\_) + 10 - \_\_$

$y = 2(x^2 + 6x + \_\_) + 10 - (2 \cdot \_\_)$

$\left(\frac{6}{2}\right)^2 = 9$

$y = 2(x^2 + 6x + 9) + 10 - (2 \cdot 9)$

$y = 2(x+3)^2 - 8$

Vertex $(-3, -8)$

AOS $x = -3$

Focus $(h, k + 1/4a)$

$(-3, -8 + 1/4(2))$

$(-3, -7.875)$

directrix $y = k - 1/4a$

$y = -8 - 1/4(2)$

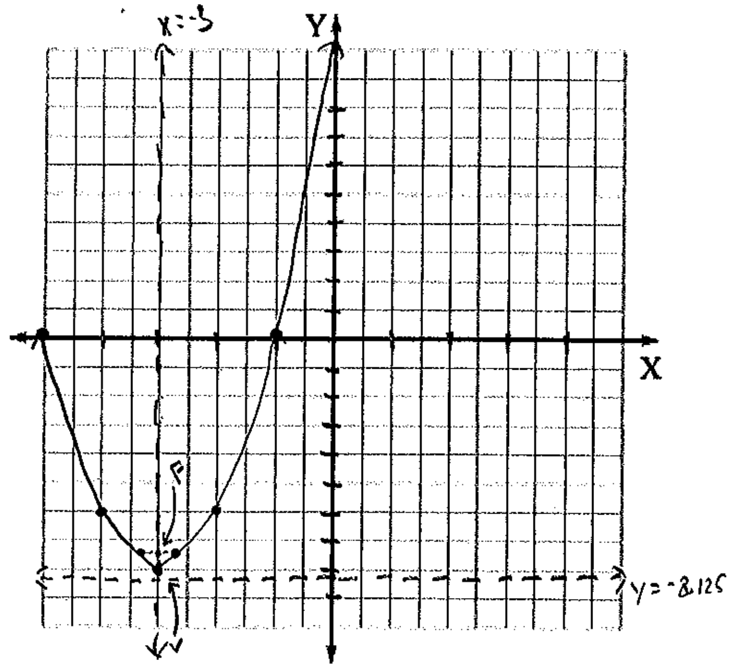
$y = -8.125$

$\underline{LR} = \left|\frac{1}{a}\right|$

$= \frac{1}{2}$

$\underline{\text{LR End points}}$ $\left|\frac{1}{2a}\right|$

$\left|\frac{1}{2(2)}\right| = \frac{1}{4}$ $(-3.25, -7.875)$ $(-2.75, -7.875)$



b) $x = -3y^2 + 12y - 4$

$x = (-3y^2 + 12y + \_\_) - 4 - \_\_$

$x = -3(y^2 - 4y + \_\_) - 4 - (-3 \cdot \_\_)$

$x = -3(y^2 - 4y + 4) - 4 - (-3 \cdot 4)$

$x = -3(y-2)^2 + 8$

Vertex $(8, 2)$

AOS $y = 2$

Focus $(h + 1/4a, k)$

$(8 + 1/4(-3), 2)$

$(7.917, 2)$

directrix: $x = h - 1/4a$

$x = 8 + 1/12$

$x = 8.083$

$LR = \left|\frac{1}{-3}\right|$

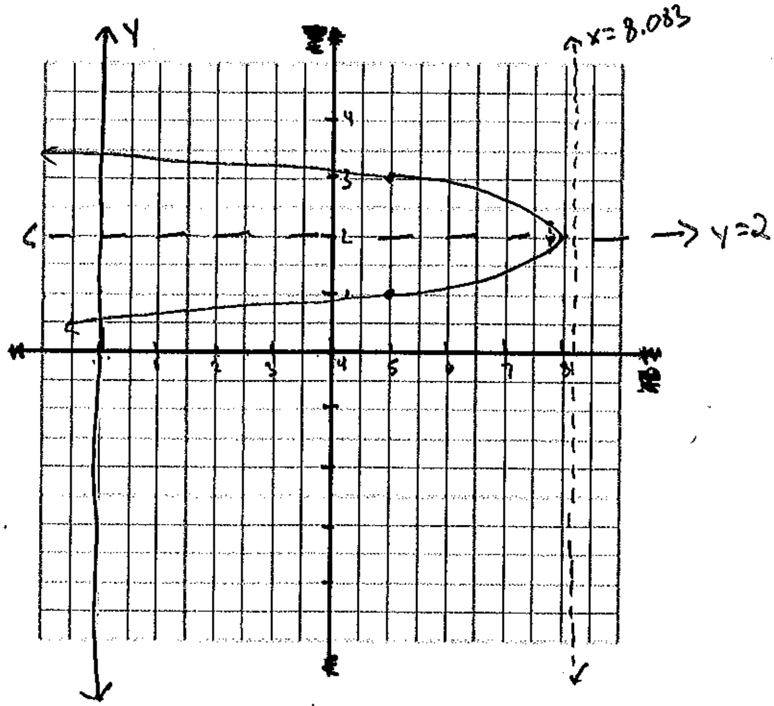
$= \frac{1}{3}$

$\underline{\text{LR endpoints}}$ $\left|\frac{1}{2a}\right|$

$\left|\frac{1}{-6}\right| = \frac{1}{6}$

$2 \pm .167$ $(7.917, 2.167)$

k $(7.917, 1.833)$



y	$-3(y-2)^2+8$	x	(x, y)
4			
3	$-3(3-2)^2+8$	5	(5,3)
2		8	(8,2)
1	$-3(1-2)^2+8$	5	(5,1)
0	$-3(0-2)^2+8$	-4	(-4,0)

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Ex. 2 Given the vertex and another piece of information, write the equation of the parabola.

a) vertex (0, 1) and focus (0, 5)

SAME so $y=$ parabola

$y=a(x-h)^2+k$

$y=a(x-0)^2+1$

Focus $(0,5)=(h,\ k+1/4a)$

$5=1+1/4a$

$4=1/4a$

$16a=1$

$a=1/16$

$y=\frac{1}{16}(x-0)^2+1$

$y=\frac{1}{16}x^2+1$

b) vertex (2, 5) and directrix y=8

$y=a(x-h)^2+k$

$y=a(x-2)^2+5$

directrix $y=k-1/4a$

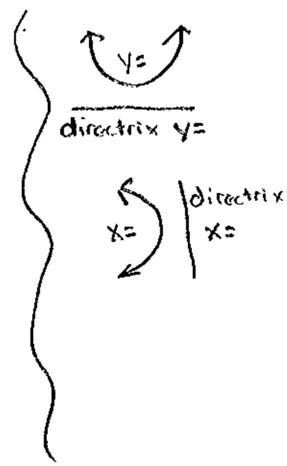
$8=5-\frac{1}{4}a$

$3=-\frac{1}{4}a$

$-12a=1$

$a=-\frac{1}{12}$

$y=-\frac{1}{12}(x-2)^2+5$



c) vertex (4, -2) and point (9, -8); opens R/L

$x=a(y-k)^2+h$

$x=$ parabola

$x=a(y+2)^2+4$

$9=a(-8+2)^2+4$

$5=36a$

$a=5/36$

$x=\frac{5}{36}(y+2)^2+4$

d) vertex (-7, 4) and AOS is x =-7, LR = 6 and a<0

$y=a(x-h)^2+k$

$y=a(x+7)^2+4$

$LR=\left|\frac{1}{a}\right|$

$6=\frac{1}{a}$

$6a=1$

$a=-\frac{1}{6}$

$y=-\frac{1}{6}(x+7)^2+4$