1.5 Inverse Functions

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7:03 AM

Inverse of a Function

Interchanging the domain values (x) with the range values (y) of a function.

Use the super script of $^{-1}$ $\quad f^{-1}$ means inverse function

$f(x) = \{(-3,4)\ (-2,1)\ (0,-3)\ (9,4)\}$
$f(x)^{-1} = \{(4,-3)\ (1,-2)\ (-3,0)\ (4,9)\}$

Steps to finding the Inverse of a Function:

1) Change f(x) into y
2) Switch x and y
3) Solve for y
4) Change y to $f^{-1}(x)$ {This denotes the inverse of f(x)}

Ex.1 Determine the inverse of each function:

a) $f(x) = \dfrac{3x-1}{7}$

$y = \dfrac{3x-1}{7}$

$x = \dfrac{3y-1}{7}$

$7x = 3y-1$

$3y = 7x+1$

$y = \dfrac{7x+1}{3}$

$f^{-1}(x) = \dfrac{7x+1}{3}$

b) $f(x) = \sqrt{x^2-1}$

$y = \sqrt{x^2-1}$

$x = \sqrt{y^2-1}$

$(x)^2 = \left(\sqrt{y^2-1}\right)^2$

$x^2 = y^2-1$

$y^2 = x^2+1$

$\sqrt{y^2} = \sqrt{x^2+1}$

$y = \pm\sqrt{x^2+1}$

$f^{-1}(x) = \pm\sqrt{x^2+1}$

c) $f(x) = \dfrac{5}{x-2}$

$y = \dfrac{5}{x-2}$

$(y-2)\ x = \dfrac{5}{y-2}\ (y-2)$

$x(y-2) = 5$

$y-2 = \dfrac{5}{x}$

$y = \dfrac{5}{x}+2$

$f^{-1}(x) = \dfrac{5}{x}+2$

Now that you can find the inverse of a function, you can verify that the functions are functions.

$f(x) = \dfrac{3x-1}{7}$ and $f^{-1}(x) = \dfrac{7x+1}{3}$; find $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$

$f(x) = \frac{3x-1}{7}$ and $f^{-1}(x) = \frac{7x+1}{3}$; find $(f \circ f^{-1})(x)$ and $(f^{-1} \circ f)(x)$

$(f \circ f^{-1})(x) = \frac{3\left(\frac{7x+1}{3}\right) - 1}{7}$

$= \frac{7x+1-1}{7}$

$= x$

$(f^{-1} \circ f)(x) = \frac{7\left(\frac{3x-1}{7}\right) + 1}{3}$

$= \frac{3x-1+1}{3}$

$= x$

* The composition of a function and its inverse is always X *

f and f^{-1} are reflections over the y-axis

Ex.2 Verify that the functions are inverses.

a) $f(x) = 2x^3 - 1$ $\quad g(x) = \sqrt[3]{\frac{x+1}{2}}$

$(f \circ g)(x) = 2\left(\sqrt[3]{\frac{x+1}{2}}\right)^3 - 1$

$= \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1$

$= x + 1 - 1$

$= x$

Yes, functions are inverses!

$(g \circ f)(x) = \sqrt[3]{\frac{(2x^3-1)+1}{2}}$

$= \sqrt[3]{\frac{2x^3}{2}}$

$= \sqrt[3]{x^3}$

$= x$

b) $f(x) = 2x+5$ $\quad g(x) = 5x+2$

$y = 2x+5$

$x = 2y+5$

$x-5 = 2y$

$y = \frac{x-5}{2}$

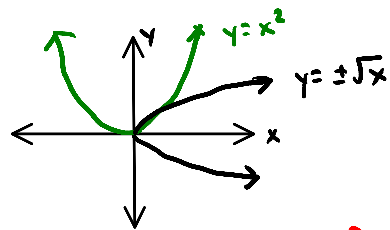
2 options

1) Find $(f \circ g)(x)$ and $(g \circ f)(x)$

2) Find the inverse of $f(x)$.

The Inverse of a function MAY or MAY NOT be a function itself.

ex: $y = x^2$ is a function; however, it's inverse of $y = \pm\sqrt{x}$ is not a function



Does not pass VLT

Passes the VLT

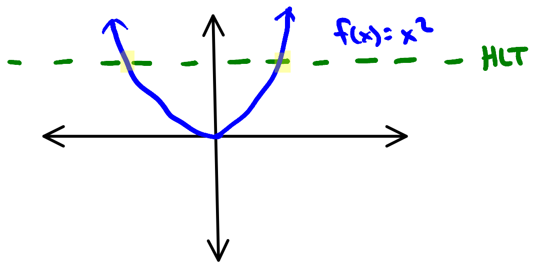
$y = \frac{5}{x-2}$ and $y^{-1} = \frac{5}{x} + 2$ are inverses

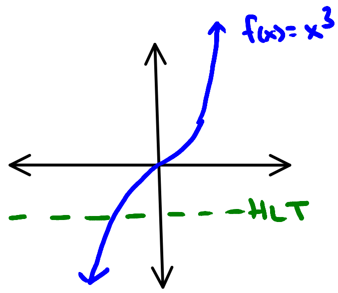
and both are functions (pass vertical line test).

One-to-One Function

When the inverse of a function is also a function.
The original function is said to be a One-to-One function.

Use the Horizontal line test on the original function to see if its inverse is a function.





The inverse of $f(x)=x^2$ is not a function, and $f(x)=x^2$ is not one-to-one function.

The inverse of $f(x)=x^3$ is a function and $f(x)=x^3$ is a One-to-one function.