1.4 Combinations of Functions

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9:23 AM

Operations of Functions

Sum: $(f+g)(x) \longrightarrow f(x) + g(x)$

Difference: $(f-g)(x) \longrightarrow f(x) - g(x)$

Product: $(f \cdot g)(x) \longrightarrow f(x) \cdot g(x)$

Quotient: $(f/g)(x) \longrightarrow \dfrac{f(x)}{g(x)}$; $g(x) \neq 0$ *must find where denominator is undefined.*

Ex.1 Perform the following operations given

$f(x) = 2x+1 \qquad g(x) = x^2+2x-1$

a) $(f+g)(x)$

$(2x+1) + (x^2+2x-1)$

x^2+4x

b) $(f-g)(x)$

$(2x+1) - (x^2+2x-1)$

$2x+1-x^2-2x+1$

$-x^2+2$

c) $(f \cdot g)(x)$

$(2x+1)(x^2+2x-1)$

$2x^3+4x^2-2x+x^2+2x-1$

$2x^3+5x^2-1$

d) $(g/f)(x)$

$\dfrac{x^2+2x-1}{2x+1}$; $x \neq -\frac{1}{2}$

$2x+1=0$

$x=-\frac{1}{2}$

e) $3(f(x))$

$3(2x+1)$

$6x+3$

$f(x) = \sqrt{x+2} \qquad g(x) = \sqrt{3-x}$

f) $(f-g)(x)$

$(\sqrt{x+2}) - (\sqrt{3-x})$

$\sqrt{x+2} - \sqrt{3-x}$

g) $(f \cdot g)(x)$

$\sqrt{-x^2+x+6}$

h) $\left(\frac{f}{g}\right)(x)$

$\dfrac{\sqrt{x+2}}{\sqrt{3-x}} \cdot \dfrac{\sqrt{3-x}}{\sqrt{3-x}} = \dfrac{\sqrt{-x^2+x+6}}{3-x}$; $x \neq 3$

Compositions of Functions

$(f \circ g)(x) = f[g(x)]$

FYI

This is the only time you work

(open circle, not an "O". It means composite

from **RIGHT** to **LEFT**.

The domain of $f \circ g$ is the set of all x in the domain g such that $g(x)$ is the domain of $f(x)$.

Range (y-values) of $g(x)$ are the domain values of $f \circ g$.

Ex. 2 Find the compositions of the given functions.

$f(x) = 2x+1$ $\quad$ $g(x) = x^2+2x-1$

a) $(f \circ g)(x) = f[g(x)]$
$= 2(x^2+2x-1)+1$
$= 2x^2+4x-2+1$
$(f \circ g)(x) = 2x^2+4x-1$

b) $(g \circ f)(x) = g[f(x)]$
$= (2x+1)^2 + 2(2x+1) - 1$
$= 4x^2+4x+1+4x+2-1$
$g[f(x)] = 4x^2+8x+2$

Ex. 3 $f(x) = x^2$ $\quad$ $g(x) = 5x$ $\quad$ $h(x) = x+4$

a) $(f \circ g)(x) = f[g(x)]$
$= (5x)^2$
$(f \circ g)(x) = 25x^2$

b) $(g \circ h)(-3) = g[f(x)]$
$h(-3) = (-3)+4 \rightarrow 1$
$g(1) = 5(1) = 5$
$(g \circ h)(-3) = 5$

c) $[f \circ (g \circ h)](4)$

$h(4) = (4)+4 = 8$
$g(8) = 5(8) = 40$
$f(40) = (40)^2$
$[f \circ (g \circ h)](4) = 1600$

Ex. 4 Does $(f \circ g)(x) = (g \circ f)(x)$?

a) $f(x) = 2x+3$ $\quad$ $g(x) = \frac{1}{2}(x-3)$

$(f \circ g)(x) = \cancel{2}\left(\frac{1}{2}(x-3)\right)+3$ $\quad$ $(g \circ f)(x) = \frac{1}{2}\left((2x+3)-3\right)$

$= x - 3 + 3$

$= x$

$= \frac{1}{2}(2x)$

$= x$

Yes

Since both compositions equal x, then yes. Identity Functions

b) $f(x) = \frac{1}{3}x - 3$ $\quad g(x) = 3x + 1$

$(f \circ g)(x) = \frac{1}{3}(3x+1) - 3$

$= x + \frac{1}{3} - 3$

$= x - \frac{8}{3}$

No, not equal.

Ex. 5 Find two functions f and g such that $(f \circ g)(x)$ is equal to $h(x)$.

a) $h(x) = \sqrt[3]{(x^2 - 4)}$

b) $h(x) = \frac{1}{x+2}$