Notes 1.3 Shifting, Reflecting, and Stretching Graphs

Horizontal & Vertical Shifts of Functions:

Horizontal Shifts

Right shift is when **(x – c)**
Left shift is when **(x + c)**

> This kind of shift occurs when a *constant* is **Added/Subtracted** **<u>INSIDE</u>** a grouping symbol: $(x), \sqrt{x}, e^x, |x|, \log(x), \sin(x)$.

This shift is sometimes confusing for students because the shift is opposite of the sign in the grouping symbol.

$f(x) = (x-4)^3$ shifts R 4

$f(x) = \sqrt{x+3}$ shift left 3

$f(x) = e^{x+2}$ shifts left 2

Vertical Shifts

Up shift when **+ c**
Down shift when **- c**

> This kind of shift occurs when a *constant* is **Added/Subtracted** to the **<u>END</u>** of a function.

$y = \ln(x) + 3$ shifts up 3

$f(x) = |x| - 2$ shifts down 2

Reflections of a Function Over an Axis:

Reflected over the **x-axis** -ax
Reflected over the **y-axis** (-ax)

> The negative sign is inside the ( )

$f(x) = -x^2$ reflects over x-axis

$f(x) = \sqrt{-x}$ reflects over the y-axis

Vertical Stretch or Compression of Functions:

horizontal
~~Vertical Stretch (aka horizontal~~ *compression*)

coefficient > 1 ie. $\underline{3}x^2$

> The graph becomes **NARROWER.**

$f(x) = 3\sin(x)$ $f(x) = \frac{3}{2}x$

Vertical Compression (*aka horizontal stretch*)

0< coefficient > 1 ie. $\frac{1}{2}x^2$

> The graph becomes **WIDER.**

$f(x) = \frac{9}{13}x^2$

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Ex. 1 Compare the graph of the function with its *parent graph*.

a) $f(x) = x^3 + 5$

Parent Graph $y = x^3$

The graph shifts 5 units up.

b) $y = e^{x-2} + 3$

Parent Graph $y = e^x$

The graph shifts 2 units right and 3 units up.

c) $f(x) = -\sqrt{x-3} - 2$

Parent Graph $y = \sqrt{x}$

The graph reflects over the x-axis, shifts 3 units right, and 2 units down.

d) $f(x) = \frac{2}{5}|-x|$

Parent Graph $y = |x|$

The graph is vertically compressed and is reflected over the y-axis.

e) $f(x) = 7\sin(x)$

Parent graph $y = \sin x$

The graph is vertically stretched.

Ex. 2 Write a function with the given information. { Start with the parent graph }

a) An absolute value function that is reflected over the x-axis, has a vertical compression, shifts left 10, and down 6 units.

$y = |x| \rightarrow y = -\frac{1}{2}|x+10| - 6$

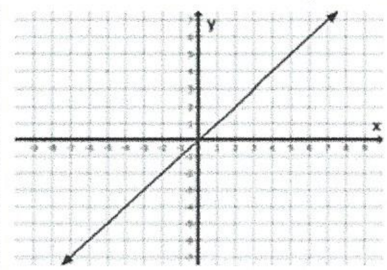
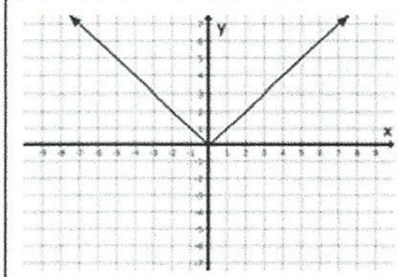
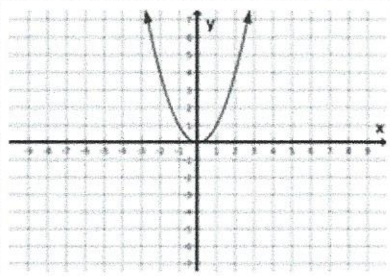
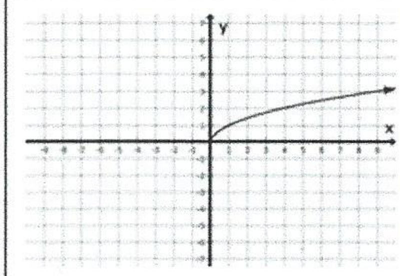
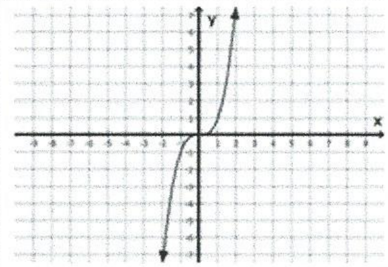
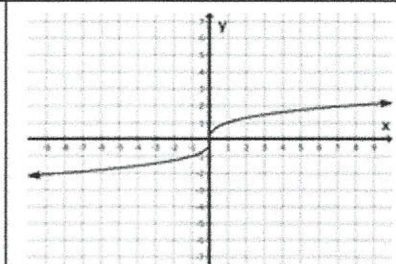
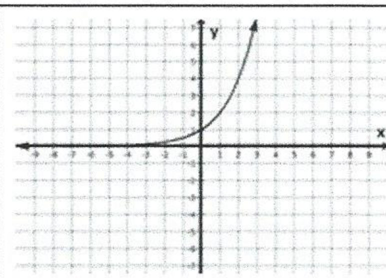
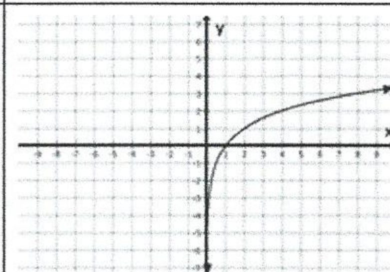
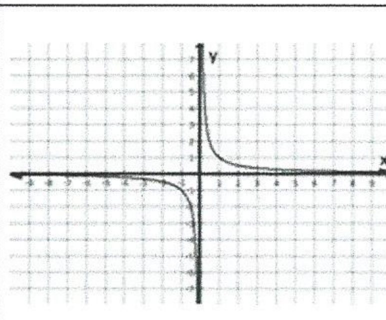
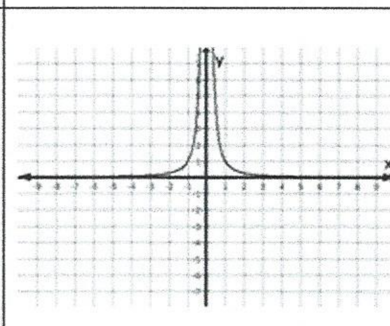
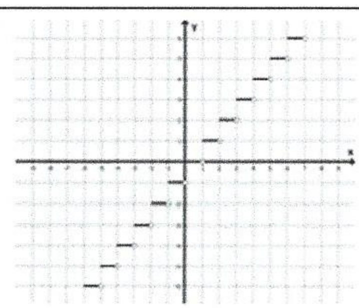
b) A log function that is vertically stretched, shifts right 2, and up 4 units.

$y = \log(x) \rightarrow y = 2\log(x-2) + 4$

c) An exponential function that is reflected over the y-axis, shifts right 3, and down 2 units.

$y = e^x \rightarrow y = e^{(-x-3)} - 2$

Parent Functions & Basic Information

Parent Function	Graph	Parent Function	Graph
$y=x$ **Linear**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=\lvert x\rvert$ **Absolute Value**, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$	
$y=x^2$ **Quadratic**, Even Domain: $(-\infty,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to-\infty,\ y\to\infty$ $x\to\infty,\ y\to\infty$		$y=\sqrt{x}$ **Radical**, Neither Domain: $[0,\infty)$ Range: $[0,\infty)$ End Behavior: $x\to\infty,\ y\to\infty$	
$y=x^3$ **Cubic**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=\sqrt[3]{x}$ **Cube Root**, Odd Domain: $(-\infty,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=b^x,\ b>1$ **Exponential**, Neither Domain: $(-\infty,\infty)$ Range: $(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to\infty$		$y=\log_b(x),\ b>1$ **Log**, Neither Domain: $(0,\infty)$ Range: $(-\infty,\infty)$ End Behavior: $x\to0^+,\ y\to-\infty$ $x\to\infty,\ y\to\infty$	
$y=\frac{1}{x}$ **Rational (Inverse)**, Odd Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(-\infty,0)\cup(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to0$		$y=\frac{1}{x^2}$ **Rational (Inverse Squared)**, Even Domain: $(-\infty,0)\cup(0,\infty)$ Range: $(0,\infty)$ End Behavior: $x\to-\infty,\ y\to0$ $x\to\infty,\ y\to0$	
$y=\text{int}(x)=[x]$ **Greatest Integer**, Neither Domain: $(-\infty,\infty)$ Range: $\{y: y\in\mathbb{Z}\}$ (integers) End Behavior: $x\to-\infty,\ y\to-\infty$ $x\to\infty,\ y\to\infty$		$y=C$ ($y=2$ in the graph) **Constant**, Even Domain: $(-\infty,\infty)$ Range: $\{y: y=C\}$ End Behavior: $x\to-\infty,\ y\to C$ $x\to\infty,\ y\to C$