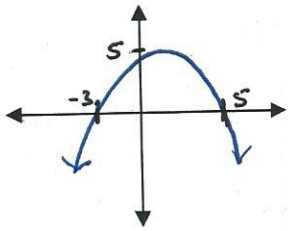
1.2 Graphs of Functions

The Graph of a Function "f" is a collection of ordered pairs (x, f(x)) such that X is the domain of "f".

Ex. 1 Find the domain and range of each function; write them in inequality notation and interval notation.

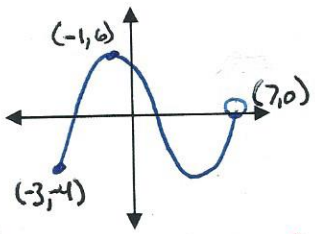
a)



Domain: $-\infty < x < \infty$ $(-\infty, \infty)$

Range: $-\infty < y \leq 5$ $(-\infty, 5]$

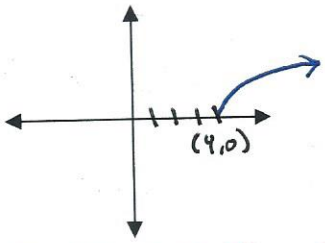
b)



D: $-3 \leq x < 7$ $[-3, 7)$

R: $-4 \leq y \leq 6$ $[-4, 6]$

c)



D: $4 \leq x < \infty$ $[4, \infty)$

R: $0 \leq y < \infty$ $[0, \infty)$

d) $f(x) = \sqrt{2x+10}$ (even index)

$2x + 10 \geq 0$

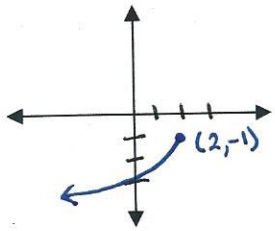
$x \geq -5$

$f(-5) = \sqrt{2(-5)+10} = \sqrt{0} = 0$

Domain: $-5 \leq x < \infty$ $[-5, \infty)$

Range: $0 \leq y < \infty$ $[0, \infty)$

e)



D: $-\infty < x \leq 2$ $(-\infty, 2]$

R: $-\infty < y \leq -1$ $(-\infty, -1]$

Interval Notation

uses [ ] and ( ) to represent inequalities, domain, and range.

[or] - for included values, closed dots; $\leq$, $\geq$

(or) - for excluded values, open dots; $<$, $>$

"or" is $\cup$ means "union".

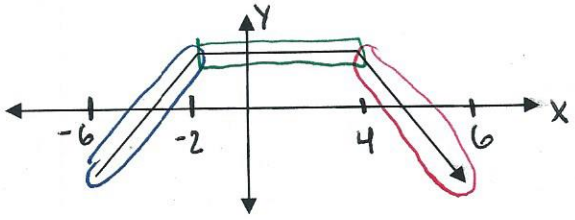
$-\infty$ negative infinity

∞ positive infinity

Increasing, Decreasing, and Constant Functions:

The more you know about a graph the more you know about the function.

- When you analyze a graph, move from the LEFT to the RIGHT.
- When stating intervals of INCREASING, DECREASING, or CONSTANT, use the x-values.



Increasing: $[-6, -2)$
from $x = -6$ to $x = -2$

Constant: $(-2, 4)$
from $x = -2$ to $x = 4$

Decreasing: $(4, \infty)$
from $x = 4$ to $x = \infty$

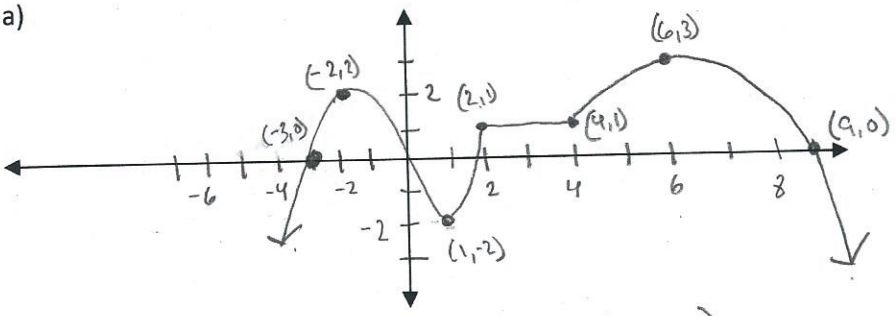
1.2 Graphs of Functions

Definitions:

- A function, f, is INCREASING on an interval if, for any x_1 and x_2 in the interval $x_1 < x_2$ implies $f(x_1) < f(x_2)$
- A function, f, is DECREASING on an interval if, for any x_1 and x_2 in the interval $x_1 < x_2$ implies $f(x_1) > f(x_2)$
- A function, f, is Constant on an interval if, for any x_1 and x_2 in the interval $f(x_1) = f(x_2)$

Ex. 2 Determine where the graph of the function is increasing, decreasing, constant, the domain, and range.

a)



* MUST LOOK AT x-values (domain).

Not ordered pairs =

Increases: $(-\infty, -2)$ $(1,2)$ $(4,6)$

Decreasing: $(-2,1)$ $(6,\infty)$

Constant: $(2,4)$

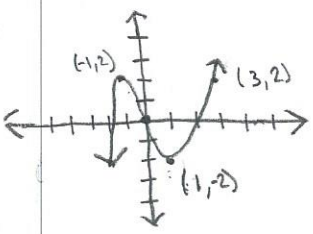
Domain: $(-\infty, \infty)$ Range: $(-\infty, 3]$

b) For $f(x) = x^3 - 3x$, find:

Increasing: $(-\infty,-1)$ $(1,\infty)$ Domain: $(-\infty,\infty)$

Decreasing: $(-1,1)$ Range: $(-\infty,\infty)$

Constant: N/A



Relative Maximum and Minimum Values

* the point at which a function changes from increasing to decreasing or decreasing to increasing; turning point

Relative Minimum: "valleys" of the function. (not always the lowest possible point)

Relative Maximum: "hills" of the function. (not always the highest possible point)

Can have multiple relative max or mins

Ex. 3 State the relative maximum(s) and minimum(s)

a) Use example **2A**

Max: $(-2,2)$
$(6,3)$ * Absolute Max

Min: $(1,-2)$

b) Use example **2B**

Max: $(-1,2)$

Min: $(1,-2)$

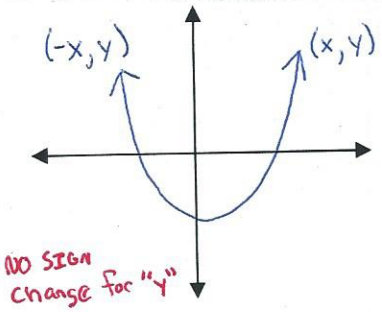
You can find the relative max/min using your calculator (Ti-83+ or ↑)

2ND Trace #4 max
#3 min

Must first plot graph under Y=.

1.2 Graphs of Functions

Even and Odd Functions:



Symmetric to the y-axis

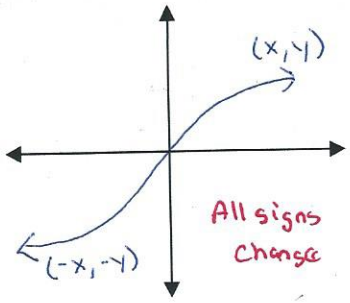
Even Function

Test: f(-x) = f(x)

$g(x) = 2x^2$

$g(-x) = 2(-x)^2$

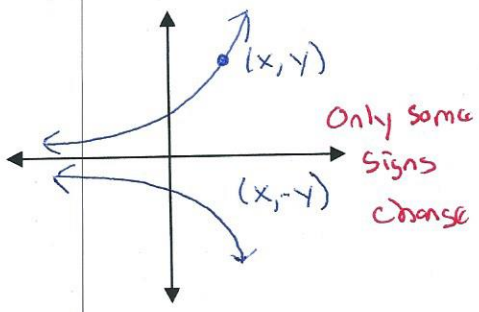
$= 2x^2$



Symmetric to the origin

Odd Function

Test: f(-x) = -f(x)



Symmetric to the x-axis

Neither

Do not have a Function!

Ex 4. Test for what type of function: even, odd, or neither.

a) $f(x) = |x|$

$f(-x) = |-x|$

$f(-x) = x$

Even

(b) $f(x) = x^2 - 5$

$f(-x) = (-x)^2 - 5$

$f(-x) = x^2 - 5$

Even

(c) $g(x) = x^3 - 5$

$g(-x) = (-x)^3 - 5$

$g(-x) = -x^3 - 5$

Some change, not all

Neither

(d) $h(x) = 5x^3 - x$

$h(-x) = 5(-x)^3 - (-x)$

$h(-x) = -5x^3 + x$

Odd

A function's graph can show you whether it is even/odd/or neither! When in doubt, graph it out!

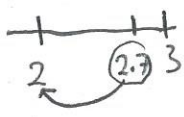
Greatest Integer Function: $[\![x]\!]$ $\lfloor x \rfloor$

- A step function. (Floor Function)
- The closest you can get to a number without going over.
- denoted by: $[\![x]\!]$

$\lfloor 2.5 \rfloor = 2$

$\lfloor 2.3 \rfloor = 2$

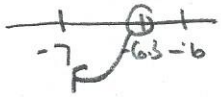
$[\![2.7]\!] = 2$



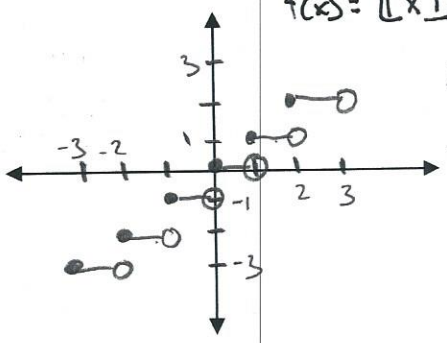
* will round down to the nearest integer *

$\lfloor -6.3 \rfloor = -7$

$\lfloor -2.5 \rfloor = -3$



$f(x) = [\![x]\!]$



$f(0) = [\![0]\!] = 0$

$f(0.5) = [\![0.5]\!] = 0$

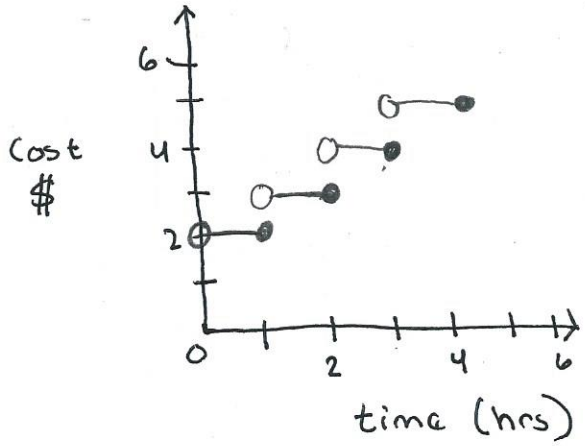
$f(-1.5) = [\![-1.5]\!] = -2$

The range of $f(x) = [\![x]\!]$ is the set of all Integers!

1.2 Graphs of Functions

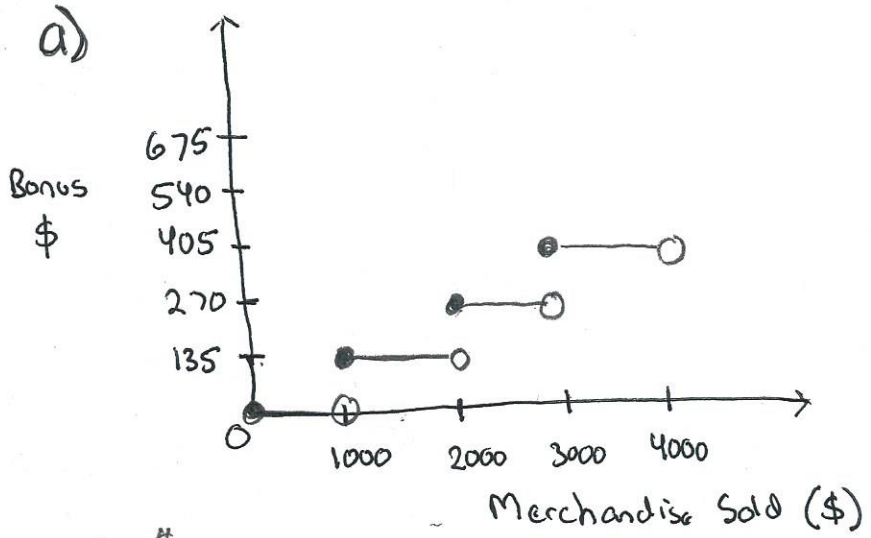
Ex. 5

a) A parking garage charges $2 for the first hour and $1 each additional hour. Graph this function.



b) A sales woman earns a $135 bonus for every $1000 worth of merchandise she sells. (a) graph the function (b) What bonus would she earn for selling $2700 worth of merchandise?

a)



b) $270