1.1 Functions

Wednesday, January 21, 2015
10:22 AM

Relation: is a set of ordered pairs that represent different quantities and are related by a rule.

Function: is a relation where each x-value in the domain is assigned to One and Only One y-value in the range.

To be a function, NO DOMAIN VALUES CAN REPEAT!

Domain
The set of INPUTS; indepent variable and is usually x-values.

Range
The set of outputs; dependent variable and is usually y-values.

Characteristic of Functions

1) Each element in the domain must be matched with an element in the range.

2) Some elements in the range may not be matched with any element in the domain.

3) Two or more elements in the domain may be matched up with the same element in the range.

ex:

Time	Temp
1	88
2	90
3	89
4	88
5	86
6	86

This is a function!

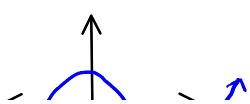
Ex. 1

Determine whether the description represents "y" as a function of "x".

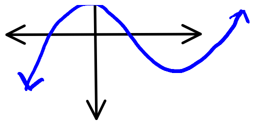
a)

input	2	3	4	2	5
output	11	8	5	10	1

Not a function b/c the input value 2 repeats.

b) 

Is a function, passes the vertical line test.



vertical line test.

IF $\underline{y}$ is a function of $\underline{\underline{x}}$; then the independent variable is x and the dependent is y.

$y = 8x+3$ $\qquad$ $\underbrace{f(x)}_{\text{function notation}} = 8x+3$

Ex.2

Which equation represents "y" as a function of "x"?

a) $x^2 + y = 1$

$y = -x^2 + 1$

This is a function, where y is a function of x.

b) $-x + y^2 = 1$

$y^2 = x+1$

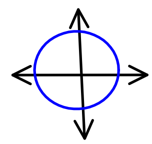
$\sqrt{y^2} = \sqrt{x+1}$

$y = \pm\sqrt{x+1}$

$x = y^2 - 1$

This $\pm$ indicates that for a given value of x, there corresponds two values of y. "y" is not a function of x.

c)



Not a function

Function Notion

$f(x)$; read as "f of x"

$f(x)$ is the value of "f" at "x".

"f" is the name of the function.

Ex.3

Evaluate the function for the given value.

$g(x) = -x^2 + 4x + 1$

a) $g(2)$

$g(2) = -(2)^2 + 4(2) + 1$

$g(2) = 5$

b) $g(t)$

$g(t) = -(t)^2 + 4(t) + 1$

$g(t) = -t^2 + 4t + 1$

c) $g(x+2)$

$g(x+2) = -(x+2)^2 + 4(x+2) + 1$

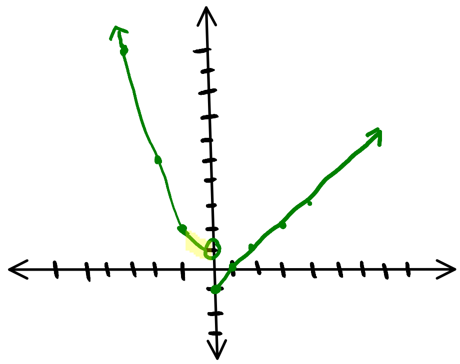
$= -(x^2 + 4x + 4) + 4x + 8 + 1$

$= -x^2 - 4x - 4 + 4x + 9$

$g(x+2) = -x^2 + 5$

Piecewise Function

Is a function defined by two or more equations over a specified domain.



$$f(x) = \begin{cases} x^2+1, & x<0 \\ x-1, & x \geq 0 \end{cases}$$

Domain $\{-3,-2,-1,0,1,2,3\}$

$x<0$

$-3,-2,-1$

$f(x)= x^2+1$

$f(-3)=(-3)^2+1=10$

$f(-2)=(-2)^2+1=5$

$f(-1)=(-1)^2+1=2$

$f(0)=(0)^2+1=1$

$x \geq 0$

$0,1,2,3$

$f(x)=x-1$

Ex. 4

Evaluate the piecewise function

$$f(x) \begin{cases} 3x^2+1, & x<-2 \\ x-3, & -2 \leq x \leq 5 \\ x^2+2 & x>5 \end{cases}$$

a) D $\{-2,-7,0,15\}$

$x<-2$

-7

$f(-7)=3(-7)^2+1$

$=148$

$-2 \leq x \leq 5$

$-2, 0$

$f(-2)=(-2)-3=-5$

$f(0)=(0)-3=-3$

$x>5$

15

$f(15)=(15)^2+2=227$

Sometimes values of the domain will be undefined and you will have to identify them; can not divide by zero or find the even root of a negative number.

ex: $f(x)=\dfrac{1}{x^2-1}$ set denominator = to 0 and solve.

$x^2-1=0$

$x^2=1$

$\sqrt{x^2}=\sqrt{1}$

$x=\pm 1$

Domain is all real #'s other than $x=\pm 1$

$f(x)=\sqrt{x}$ $x \geq 0$

radicand

Domain is all positive #'s   SAME $[0,\infty)$ Interval notation

Both of the above examples are Implied Domains

Ex.5 State the domain of each function.

a) $h(x) = \frac{1}{x-3}$

$x-3=0$

$x=3$

Domain all real #'s except $x=3$

b) $f(x) = \sqrt{6+4x}$

$6+4x \geq 0$

$4x \geq -6$

$x \geq -\frac{3}{2}$

$\mathbb{R}$ - all real numbers!

$\mathbb{R}$

Domain is all real #'s except where $x \geq -3/2$

c) $V = \frac{4}{3}\pi r^3$

Domain is all positive numbers.

d) $h(x) = 6x^2 - 8x + 6$

Domain $\mathbb{R}$

e) $\{(6,3)\ (5,8)\ (0,-6)\ (1,8)\}$

Domain $\{6, 5, 0, 1\}$

One of the basic definitions in calculus employs the ratio

$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

This is called the Difference Quotient.

Ex.6 Evaluating a Difference Quotient.

a) $f(x) = -2x + 4$ find $\frac{f(x+h) - f(x)}{h}$

* substitute "x+h" for "x" into the original function.

$$\frac{[-2(x+h)+4] - (-2x+4)}{h}$$

$$\frac{[-\cancel{2x} - 2h + \cancel{4}] + \cancel{2x} - \cancel{4}}{h} \rightarrow \frac{-2h}{h} \rightarrow -2;\ h \neq 0$$

b) $f(x) = x^2 - 4x + 7$ find $\frac{f(x+h) - f(x)}{h}$

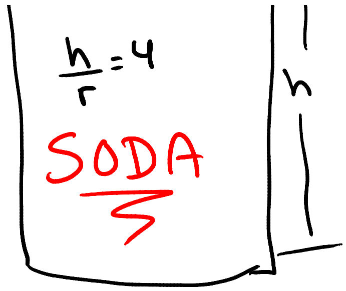
$$\frac{[(x+h)^2 - 4(x+h) + 7] - (x^2 - 4x + 7)}{h}$$

$$\frac{[\cancel{x^2} + 2xh + h^2 - \cancel{4x} - 4h + \cancel{7}] - \cancel{x^2} + \cancel{4x} - \cancel{7}}{h} \rightarrow \frac{h^2 + 2xh - 4h}{h} \rightarrow h + 2x - 4;\ h \neq 0$$

Ex.7 Applications of Functions

a) example # 6 pg 79

1) Express the volume of the can as a function of the radius.



$V=\pi r^2 h$ $\qquad$ $\frac{h}{r}=4$

$V=\pi r^2(4r)$ $\qquad$ $h=4r$

$V=4\pi r^3$

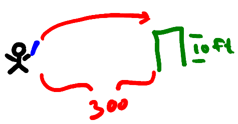
2) Express the volume of the can as a function of the height.

$V=\pi r^2 h$ $\qquad$ $\frac{h}{r}=4 \rightarrow r=\frac{h}{4}$

$V=\pi\left(\frac{h}{4}\right)^2 h \longrightarrow V=\frac{\pi h^3}{16}$

b) example #7 pg 79

$f(x)=-0.0032x^2+x+3$



1) will the ball clear a 10ft fence at 300 ft?

$f(300)=-0.0032(300)^2+(300)+3$

$=15$ ft

2) will the ball clear a 20ft fence at 300 ft?

No