3.2 Logarithmic Functions and Graphs

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9:47 AM

Logarithmic Function with Base "a"

$y=a^x$ is an exponential function and passes the "HLT"; because it passes the HLT, its inverse is also a function.

The inverse is the Logarithmic Function with base "a".

$y=\log_a x$ $x>0$ $a>0$ but $\neq 1$

Exponential Form		Logarithmic Form
$x=a^y$	⟵ SAME ⟶	$y=\log_a x$

* It is important to remember that logarithms are Exponents!! *

Ex. 1 Write each expression in exponential form

a) $\log_3 9 = 2$

$3^2=9$

b) $\log_{10}(1/100) = -2$

$10^{-2} = 1/100$

c) $\log_5 125 = 3$

$5^3=125$

Ex. 2 Write each expression in Logarithmic Form

a) $5^3=125$

$\log_5 125 = 3$

b) $27^{(1/3)}=3$

$\log_{27} 3 = 1/3$

c) $10^3=1000$

$\log_{10} 1000 = 3$

Ex. 3 Evaluate each expression (rewrite in exp. form, make same bases)

a) $\log_5 125$

$\log_5 125 = x$

$5^x = 125$

$\cancel{5}^x = \cancel{5}^3$

3

b) $\log_3 81$

$\log_3 81 = x$

$3^x = 81$

$\cancel{3}^x = \cancel{3}^4$

4

c) $\log(1/10{,}000)$

$\log_{10}(1/10{,}000) = x$

$10^x = 1/10{,}000$

$10^x = 10^{-4}$

-4

Common Logarithmic Function is a log with base 10!

* Calculator only does $\log_{10}$ (common log) $\log_{10} 5 \leftrightarrow \log 5$

<u>Ex.4</u> Use a calculator to evaluate each expression to 4 decimal places.

a) $\log_{10} 10$

1

b) $\log_{10} 8$

$.9031$

c) $2 \log 2.5$

$.7959$

d) $\log_{10} -8$

<u>undefined</u> ; Domain $(0, \infty)$

<u>Properties of Logarithms</u>

1) $\log_a 1 = 0$ b/c $a^0 = 1$

2) $\log_a a = 1$ b/c $a^1 = a$

3) $\underbrace{\log_a a^x = x \quad \text{or} \quad a^{\log_a x} = x}_{\text{Inverse Property}}$

4) If $\log_a x = \log_a y$, then $x = y$. ← One-to-one property

<u>Ex.5</u> Evaluate each expression or solve the equation.

a) $\log_8 8^2$

2

b) $900^{\log_{900}(x^2-1)}$

x^2-1

c) $\log_6 x = \log_6 4$

$x = 4$

a) $\log_8 8^2$

2

b) $900^{\log_{900}(x^2-1)}$

x^2-1

c) $\log_6 x = \log_6 4$

$x=4$

$\{4\}$

d) $\log_4 4 = x$

$4^x = 4$

$x=1$

$\{1\}$

e) $\log_4 1$

$\log_4 1 = x$

$4^x = 1$

0

Since logarithmic functions are the inverse of exponential functions:

a) domain of exp. function is the range of a log function

b) range of exp. function is the domain of a log function

$y=\log_a x$ is the inverse of $y=a^x$.

	$y=\log_a x$	$y=a^x$
Domain	$(0,\infty)$	$(-\infty,\infty)$
Range	$(-\infty,\infty)$	$(0,\infty)$

$y=\log_a x$ is only defined if $\underline{\underline{x}}$ is $\underline{\underline{\text{positive}}}$!

$a>0$, $a\neq 1$

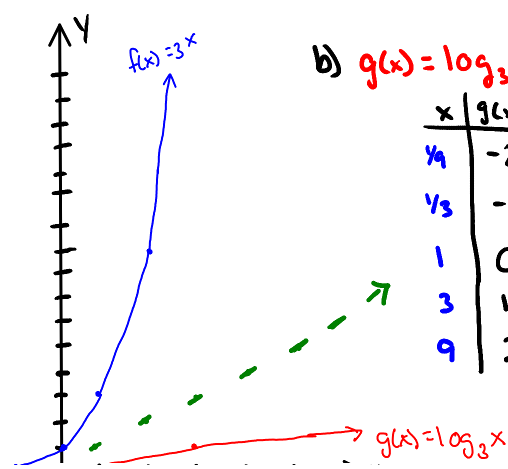
$x>0$

To sketch the graph of $y=\log_a x$ use the fact that the graphs of the inverses are REFLECTIONS of each other over the line $y=x$.

<u>Ex.6</u> Sketch the graph.

a) $f(x)=3^x$

x	$f(x)$
-2	1/9
-1	1/3
0	1
1	3
2	9

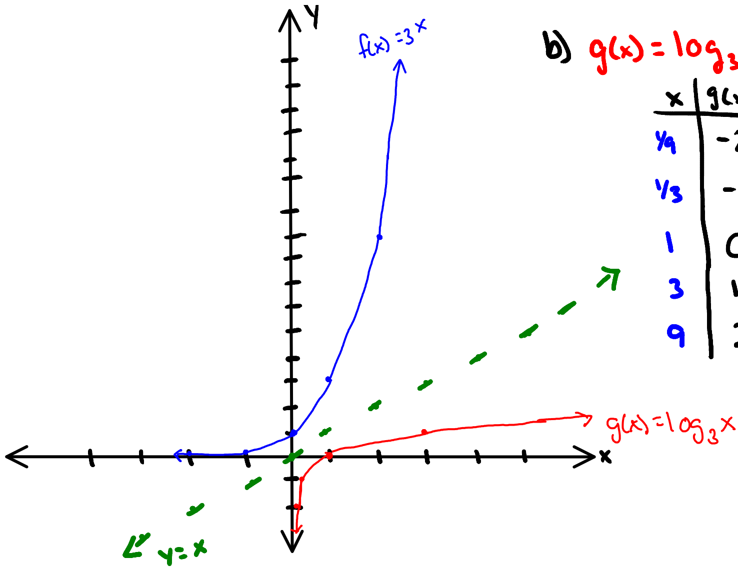


b) $g(x)=\log_3 x$

x	$g(x)$
1/9	-2
1/3	-1
1	0
3	1
9	2

a) $f(x) = 3^x$

x	$f(x)$
-2	$1/9$
-1	$1/3$
0	1
1	3
2	9



b) $g(x) = \log_3 x$

x	$g(x)$
$1/9$	-2
$1/3$	-1
1	0
3	1
9	2

From exponential to logarithmic the domain and range switch.

Graph of $y = \log_a x$ $\quad a > 1$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Intercept: x-int $(1, 0)$

Asymptote: VA $x=0$ (y-axis)

It increases

It is continuous

It is a reflection of $y = a^x$ over the line $y = x$

Log functions can be transformed just like any other function.

Ex. 7 Describe the change from the parent graph of $y = \log x$

a) $y = \log(x+6)$

Shifts 6 units left

b) $y = \log(x) - 5$

shifts 5 units down