8.5 – Trigonometric Equations and its Applications

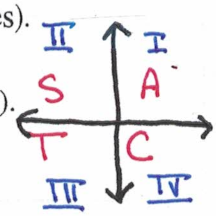
> Example: $\sin x\,(\cos x - 1) = 0$ Factored out
>
> Do NOT do → $\dfrac{\sin x\,(\cos x - 1)}{\sin x} = \dfrac{0}{\sin x}$
>
> Do do → $\sin x = 0$ and $\cos x - 1 = 0$
> Use ZPP to solve

Steps For Solving Trigonometric Equations

1.) Simplify the equation like any other equation such as…
- Get rid of parentheses by using the distributive property
- Collect like terms and isolate the trigonometric function (which contains x) on one side
- If the trigonometric function is already in factored form →
  SET EACH FACTOR = 0 and solve them! Do NOT divide out each side by a trig expression!
  (Zero Product Property)
- Cross multiply if have (or get) the equation to have a ratio (fraction) on either side of equal sign.

2.) Many equations will have MULTIPLE SOLUTIONS (ANGLES) where some (or all) don't check → so you can have… a.) 1 , 2 , 3 , or 4 solutions b.) some extraneous solution(s) c.) no solution
(solutions that do not work)

3.) Make sure you know what QUADRANTS sine, cosine, and tangent are POSITIVE and NEGATIVE. Refer to your Trig Chart/Unit Circle Sheet and where to obtain some multiple answers (angles).



**Notes:
- Your solution(s) will need to be between 0° and 360° so some angles may not "fit", any angle that doesn't "fit" this rule is considered an extraneous solution (doesn't fit/work/check).
- You are ALLOWED to add 360° to make a negative angle become positive BUT you are NOT ALLOWED to subtract 360° to make positive angle "fit" the rule.
- You must put final solution(s) in ascending order, do not include any extraneous solutions, and do not rewrite solutions that are repeated. * ALWAYS ✓ YOUR SOLUTIONS! *

Example 1: Solve each trigonometric equation. Keep your answer(s) in degrees where $0° \le x < 360°$.

a.) $\cos x + \cos x = \sqrt{2}$

$x + x = \sqrt{2}$

combine like terms

$\dfrac{2\cos x}{2} = \dfrac{\sqrt{2}}{2}$

$\cos x = \dfrac{\sqrt{2}}{2}$

$x = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$x = 45°$ What Quadrants is cos +? I, IV

I →

IV $x = 360 - 45° = 315°$

Solution(s): $45°, 315°$

b.) $3\sin x - 2 = 5\sin x - 1$

$3x - 2 = 5x - 1$

$-3\sin x$ $+1$ $-3\sin x$ $+1$

$\dfrac{-1}{2} = \dfrac{2\sin x}{2}$

$\sin x = -\frac{1}{2} \to x = \sin^{-1}(-1/2)$

What Quad is sin −? III, IV

$x = -30° + 360$

$x = 330°$ IV

III $x = 180 + 30$

$x = 210°$

Solution(s): $210°, 330°$

c.) $\tan x\left(2\sin x - \sqrt{3}\right) = 0$

$\tan x = 0$

$x = \tan^{-1}(0)$

$x = 0°, 180°$

$2\sin x - \sqrt{3} = 0$

$+\sqrt{3}$ $+\sqrt{3}$

$\dfrac{2\sin}{2} = \dfrac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2} \to x = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = 60°$ I

Quad II = $180 - 60$ $= 120°$

Solution(s): $0°, 60°, 120°, 180°$

d.) $2\tan 2x - 8\tan 2x = 6\sqrt{3}$

$2x - 8x = 6\sqrt{3}$

combine like terms

$\dfrac{-6\tan 2x}{-6} = \dfrac{6\sqrt{3}}{-6}$

$\tan 2x = -\sqrt{3} \to 2x = \tan^{-1}(-\sqrt{3})$

$2x = -60 + 360$ ← what Quad is tan −? II, IV

$2x = 300°$ IV

$x = 150°$ ← II

Solution(s): $150°, 300°$

e.) $3\cos x = 6 - 2(1 - \cos x)$

$3\cos x = 6 - 2 + 2\cos x$

$3\cos x = 4 + 2\cos x$

$-2\cos x$ $-2\cos x$

$\cos x = 4$

$x = \cos^{-1}(4)$

$x = \varnothing$

Solution(s): $\varnothing$

f.) $\dfrac{1}{\sin x} = \dfrac{2}{\sqrt{2}}$

$\dfrac{2\sin x}{2} = \dfrac{\sqrt{2}}{2}$

$\sin x = \frac{\sqrt{2}}{2} \to x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$x = 45°$ I Where is sin +? I, II

$x = 180 - 45$

II $x = 135°$

Solution(s): $45°, 135°$

g.) $(2\cos x - 1)(\sin x + 1) = 0$

$2\cos x - 1 = 0$

$+1$ $+1$

$\dfrac{2\cos x}{2} = \dfrac{1}{2}$

$\cos x = 1/2$

$x = \cos^{-1}(1/2)$

$x = 60°$ I, IV

$x = 360 - 60$

$x = 300°$

$\sin x + 1 = 0$

-1 -1

$\sin x = -1$

$x = \sin^{-1}(-1)$

$x = -90$

where is sin −?

$x = -90 + 360$

$x = 270$

Solution(s): $60°, 270°, 300°$

h.) $8\cos\left(\frac{3}{7}x\right) + 4\sqrt{3} = 0$

$-4\sqrt{3}$ $-4\sqrt{3}$

$\dfrac{8\cos\left(\frac{3}{7}x\right)}{8} = \dfrac{-4\sqrt{3}}{8}$

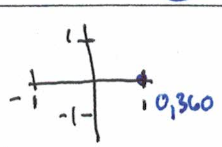
$\cos\frac{3}{7}x = -\frac{\sqrt{3}}{2}$ ← where is cos −? II, III

$\frac{3}{7}x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$\frac{3}{7}x = 150$ $\frac{3}{7}x = 210$

$x = 350°$ $x = \cancel{490°}$

Solution(s): $350°$



Example 2: **Complete each application problem involving trigonometric equations.**

a.) The tide cycle of a city on the Atlantic coast can be represented by the equation of $h = 9\sin(30^\circ t)$ where h = height of the tide in feet and t = number of hours since the last high tide. A tide is at equilibrium when it's at its normal level, halfway between its highest and lowest points. How many hours will it take for the first low tide to reach 3 feet?

$h = 9\sin(30^\circ t)$

$h = -3$

$\frac{-3}{9} = \frac{9\sin(30^\circ t)}{9}$

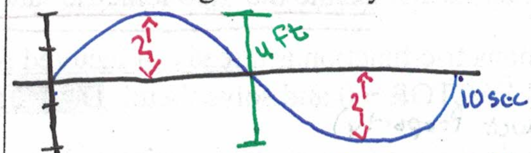
$-\frac{1}{3} = \sin(30^\circ t) \rightarrow 30^\circ t = \sin^{-1}(-\frac{1}{3})$

$30^\circ t = -19.47122063^\circ + 360^\circ$

$\frac{30^\circ t}{30^\circ} = \frac{340.52877940}{30^\circ}$

$t = 11.4$ hrs

b.) A buoy in the harbor of San Juan, Puerto Rico, bobs up and down. The distance between the highest and lowest point is 4 feet. It moves from its highest point down to its lowest point and back to its highest point every 10 seconds. How many seconds will it take for the height of the buoy to be 1.75 feet?



Amp = 2

$a = 2$

Per = 10 sec

$b = \frac{2\pi}{\text{per}}$

$b = \frac{2\pi}{10}$

$b = \frac{\pi}{5} \cdot \frac{180}{\pi}$

$b = 36^\circ$

$y = a\sin(bx \cancel{\pm c}) \cancel{\pm d}$

$y = a\sin bt$

$h = a\sin bt$

$\frac{1.75}{2} = \frac{2\sin 36^\circ t}{2}$

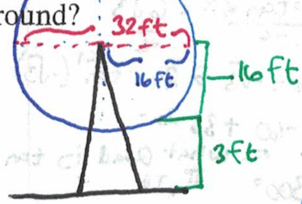
$\sin 36^\circ t = .875$

$36^\circ t = \sin^{-1}(.875)$

$\frac{36^\circ t}{36} = \frac{61.04497563}{36}$

$t = 1.7$ sec

c.) As you ride a Ferris wheel, the height that you are above the ground varies periodically as a function of time. Jason gets into a seat that is at the bottom of the Ferris wheel, he is 3 feet above the ground. The wheel has a diameter of 32 feet and it takes the wheel 18 seconds to complete one cycle. After how many seconds will it take for Jason to be 32.5 feet above the ground?



$y = a\sin(bt) \pm d$

Amp = 16 $\quad a = 16$

Per = 18 $\quad b = \frac{2\pi}{18}$

$= \frac{\pi}{9} \cdot \frac{180}{\pi}$

$= b = 20^\circ$

Vertical Shift = 16 + 3 = 19

$32.5 = 16\sin(20^\circ t) + 19$
$-19 \qquad\qquad -19$

$\frac{13.5}{16} = \frac{16\sin(20^\circ t)}{16}$

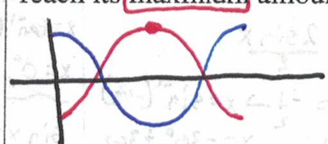
$.84375 = \sin(20^\circ t)$

$20^\circ t = \sin^{-1}(.84375)$

$20^\circ t = 57.53825503$

$t = 2.9$ sec

d.) In a certain wildlife refuge, the population of field mice can be modeled by the equation $h = 3000 - 1250\cos(120^\circ t + 4)$ where h = the number of mice and t = the number of months past March 1 of a given year. About what date will the number of mice reach its $\boxed{\text{maximum}}$ amount?



Amp

$h = 3000 - 1250\cos(120^\circ t + 4)$

vertical shift

h = max amount of mice!

h = vertical + Amp

3000 + 1250

h = 4250

$4250 = 3000 - 1250\cos(120^\circ t + 4)$
$-3000 \quad -3000$

$\frac{1250}{-1250} = \frac{-1250\cos(120^\circ t + 4)}{-1250}$

$-1 = \cos(120^\circ t + 4)$

$120^\circ t + 4 = \cos^{-1}(-1)$

$120^\circ t + 4 = 180^\circ$
$-4 \qquad -4$

$120^\circ t = 176^\circ$

$t = 1.5$ month

March 1 add 1.5 months

April 15th