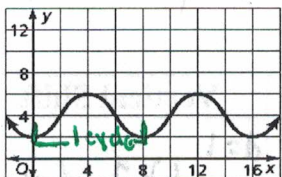
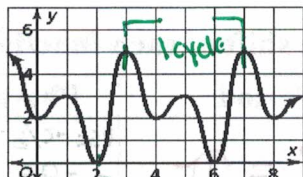
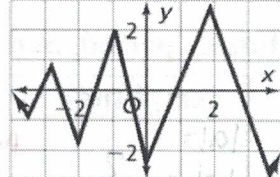
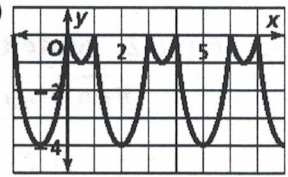


8.3 – Graphing Sine and Cosine Functions

Periodic Function and Period

- One basic property of both the sine and the cosine function → considered periodic
- **periodic function** → a function that repeats a pattern of y-values at regular intervals where one cycle equals one period.
- **period (of a periodic function)** → the horizontal length of one cycle

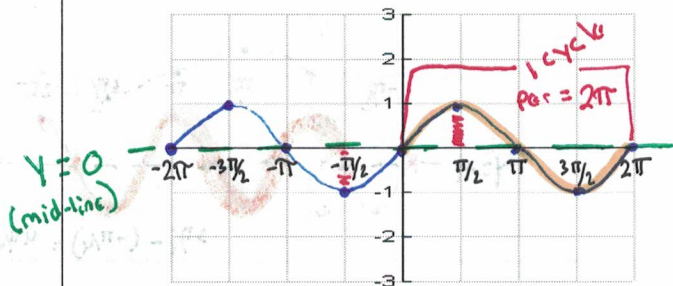
Example 1: Determine if the given function is periodic. If so, state the period.

<p>a.) </p> <p>Periodic? <u>Yes</u> No</p> <p>Period = <u>8</u> $8-0=8$</p>	<p>b.) </p> <p>Periodic? <u>Yes</u> No</p> <p>Period = <u>4</u> $7-3$</p>	<p>c.) </p> <p>Periodic? Yes <u>No</u></p> <p>Period = <u>N/A</u></p>	<p>d.) </p> <p>Periodic? <u>Yes</u> No</p> <p>Period = <u>3</u> $5-2$</p>
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Graph of Trig Function # 1 - Sine

Make a table of domain values between $\pm 2\pi$ for the function of $y = \sin(x)$ (Parent)

X	-2π	$-3\pi/2$	$-\pi$	$-\pi/2$	0
Y	0	1	0	-1	0
	$\pi/2$	π	$3\pi/2$	2π	
	1	0	-1	0	



Characteristics of the Sine Function:

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$
 Period: 2π Amplitude: 1

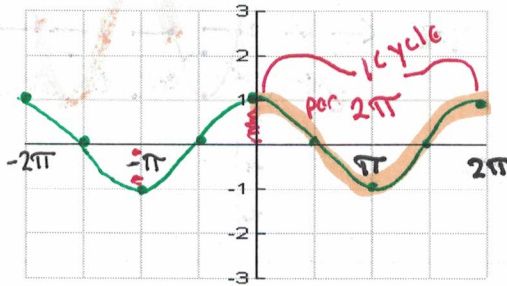
Important Part of Graph: **Domain is Positive**

- | | |
|----------------------------|---|
| 1.) <u>Start</u> $(0,0)$ | 4.) <u>Down</u> $(3\pi/2, -1)$ |
| 2.) <u>Up</u> $(\pi/2, 1)$ | 5.) <u>Up</u> $(2\pi, 0)$ ENDS |
| 3.) <u>Down</u> $(\pi, 0)$ | looks like a <u>sideway S</u> or a series of waves! |

Graph of Trig Function # 2 - Cosine

Make a table of domain values between $\pm 2\pi$ for the function of $y = \cos(x)$ (Parent)

X	-2π	$-3\pi/2$	$-\pi$	$-\pi/2$	0
Y	1	0	-1	0	1
	$\pi/2$	π	$3\pi/2$	2π	
	0	-1	0	1	

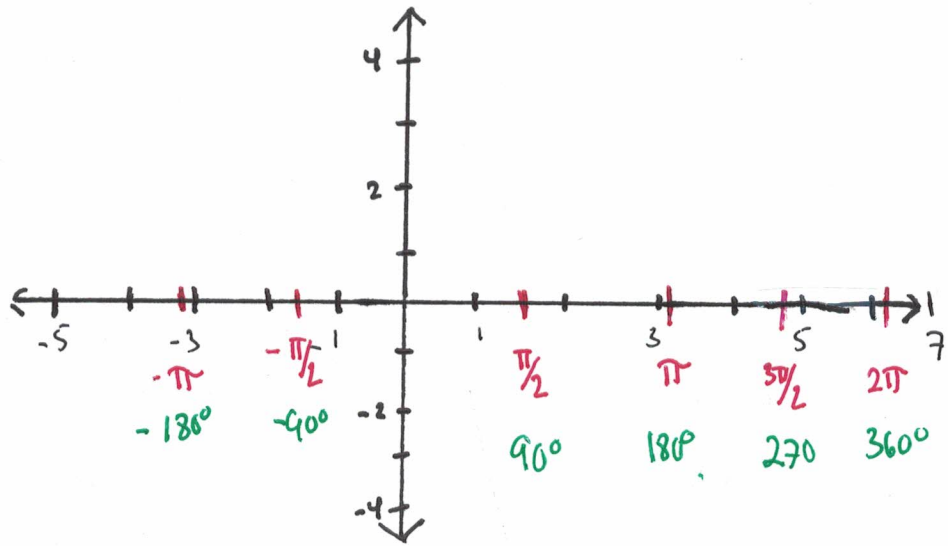


Characteristics of the Cosine Function:

Domain: $(-\infty, \infty)$ Range: $[-1, 1]$
 Period: 2π Amplitude: 1

Important Part of Graph: **Domain is Positive**

- | | |
|------------------------------|---|
| 1.) <u>Start</u> $(0,1)$ | 4.) <u>Up</u> $(3\pi/2, 0)$ |
| 2.) <u>Down</u> $(\pi/2, 0)$ | 5.) <u>Up</u> $(2\pi, 1)$ |
| 3.) <u>Down</u> $(\pi, -1)$ | looks like a <u>modified U</u> upside down bell-curve |



$y = \sin x \leftarrow \text{Parents} \rightarrow y = \cos x$
 $a=1 \quad b=1 \quad c=0 \quad d=0$

Graphing the Sine / Cosine Function: $f(x) = a \sin (bx \pm c) \pm d$ or $f(x) = a \cos (bx \pm c) \pm d$

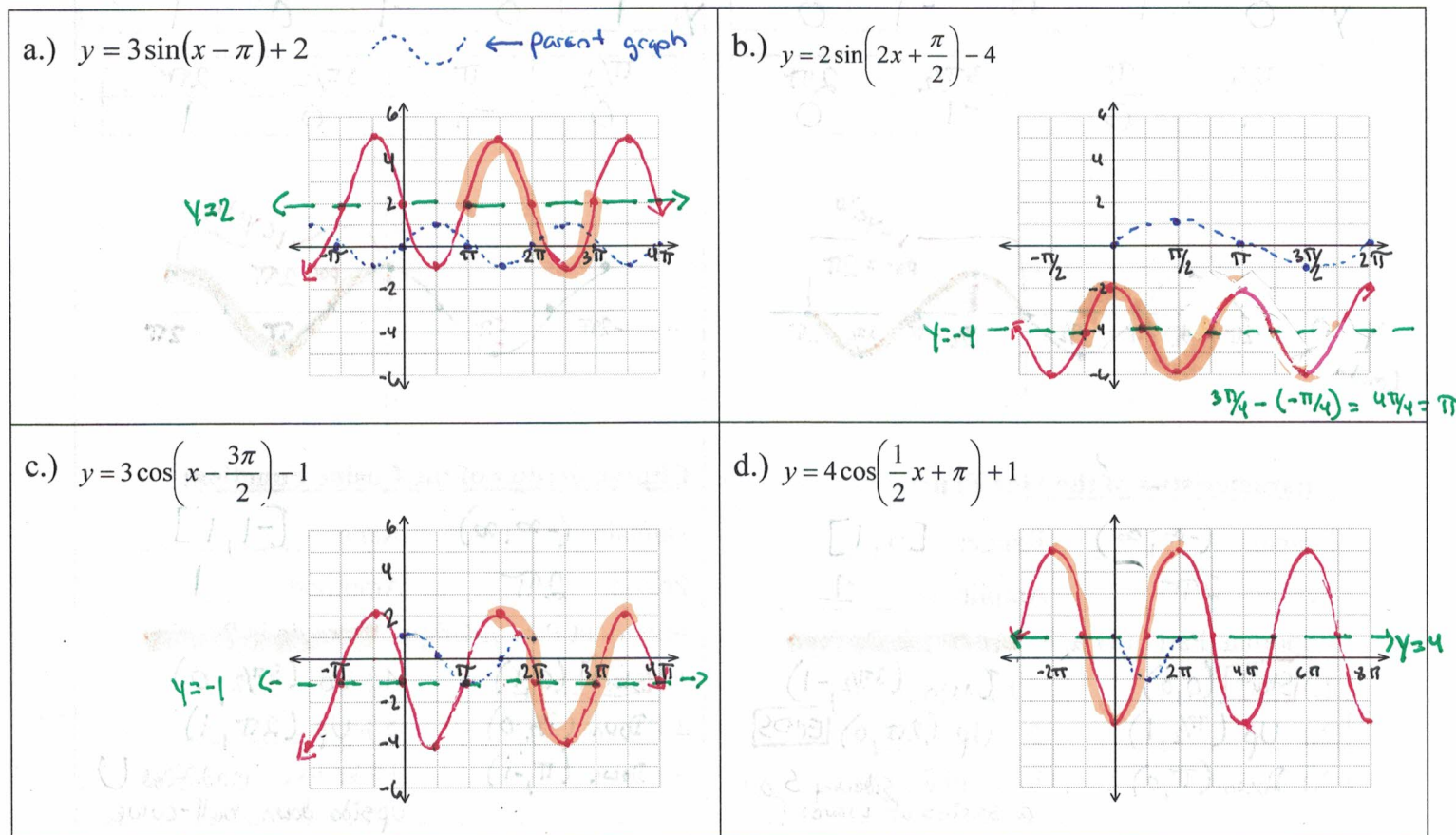
Each parameter (letter) affects the graph of $y = a \sin/\cos (bx \pm c) \pm d$ differently:

- Parameter **a** affects the range of $f(x)$ where $|a|$ is called the **AMPLITUDE** (wave height from mid-line)
- Parameter **b** affects the period of $f(x)$ where the period = $\frac{2\pi}{b}$
- Parameter **c** affects the horizontal shift of $f(x)$ where phase shift = $-\frac{c}{b}$
 - If $-\frac{c}{b} < 0$ then the graph shifts to the left
 - If $-\frac{c}{b} > 0$ then the graph shifts to the right
- Parameter **d** affects the vertical shift of $f(x)$ *** It moves the mid-line!**
 - If $d < 0$ then the graph shifts down
 - If $d > 0$ then the graph shifts up

Example 2: State the amplitude, period, and phase shift of each function.

Function	Amplitude	Period	Phase Shift	Vertical Shift
a.) $y = 3 \sin (2x) + 1$ $a=3 \quad b=2 \quad c=0 \quad d=1$	$ a =3$ $ 3 =3$	$Per = \frac{2\pi}{b}$ $= \frac{2\pi}{2} = \pi$	$PS = -\frac{c}{b}$ $= -\frac{0}{2} = 0$	$d=1$ Up 1 (mid-line $y=1$)
b.) $y = -2 \cos \left(x + \frac{\pi}{2} \right)$ $a=-2 \quad b=1 \quad c=\frac{\pi}{2} \quad d=0$	$ a =2$ $ -2 =2$	$Per = \frac{2\pi}{b}$ $= \frac{2\pi}{1} = 2\pi$	$PS = -\frac{c}{b}$ $= -\frac{(\pi/2)}{1} = -\pi/2$ (left)	$d=0$ None ($y=0$ mid-line)
c.) $y = \sin (4x - \pi) - 3$ $a=1 \quad b=4 \quad c=-\pi$	$ a =1$ $ 1 =1$	$Per = \frac{2\pi}{b}$ $\frac{2\pi}{4} = \pi/2$	$PS = -\frac{c}{b}$ $= -\frac{(-\pi)}{4} = \pi/4$ (right)	$d=-3$ Down 3 (mid-line $y=-3$)
d.) $y = \frac{1}{2} \cos \left(\frac{1}{4}x + \pi \right) + 2$ $a=1/2 \quad b=1/4 \quad c=\pi$	$ a =1/2$ $ 1/2 =1/2$	$Per = \frac{2\pi}{b}$ $= \frac{2\pi}{1/4} = 8\pi$	$PS = -\frac{c}{b}$ $= -\frac{(\pi)}{1/4} = -4\pi$ (left)	$d=2$ Up 2 (mid-line $y=2$)

Example 3: Graph each function by finding the amplitude, period, phase shift, and vertical shift.



$$3a) y = 3 \sin(x - \pi) + 2$$

$$a = 3 \quad b = 1 \quad c = -\pi \quad d = 2$$

$$\text{Amp} = |a|$$

$$= |3|$$

$$= 3$$

Represents the wave height or depth from the mid-line!

$$\text{Per} = \frac{2\pi}{b}$$

$$= \frac{2\pi}{1}$$

$$= 2\pi$$

How long one cycle takes!

$$\text{interval} = \frac{\text{per}}{4}$$

$$\text{PS} = \frac{-c}{b}$$

$$= \frac{-(-\pi)}{1}$$

$$= \pi$$

right π units.

$$vs = d$$

$$= 2$$

up 2

mid-line is now at $y = 2$

$$= \frac{2\pi}{4} = \frac{\pi}{2} \text{ (interval)}$$

This is the interval of the x-axis!

$$3b) y = 2 \sin(2x + \frac{\pi}{2}) - 4$$

$$a = 2 \quad b = 2 \quad c = \frac{\pi}{2} \quad d = -4$$

$$\text{Amp} = |a|$$

$$= |2|$$

$$= 2$$

$$\text{Per} = \frac{2\pi}{b}$$

$$= \frac{2\pi}{2}$$

$$= \pi$$

$$\text{interval} = \frac{\text{Per}}{4}$$

$$= \frac{\pi}{4}$$

x-axis should be

labeled every $\frac{\pi}{4}$!

$$\text{PS} = \frac{-c}{b}$$

$$= \frac{-(\frac{\pi}{2})}{2}$$

$$= -\frac{\pi}{4}$$

$$-\frac{\pi}{2} \cdot \frac{1}{2} = -\frac{\pi}{4}$$

left $\frac{\pi}{4}$ units

$$vs = d$$

$$= -4$$

down 4

mid-line is $y = -4$

$$\frac{\pi}{4} \quad \frac{2\pi}{4} \quad \frac{3\pi}{4} \quad \frac{4\pi}{4}$$

$$\frac{\pi}{2} \quad \pi$$

3c) $y = 3 \cos(x - 3\pi/2) - 1$

$a=3$ $b=1$ $c=-3\pi/2$ $d=-1$ $S + (\pi - x) \text{ rad} = y$ (at $S=b$ $\pi=0$ $1=d$ $S=0$)

$\text{Amp} = |a| = |3| = 3$
 $\text{Per} = 2\pi/b = 2\pi/1 = 2\pi$
 $\text{Ps} = -c/b = -(-3\pi/2) = 3\pi/2$
 $\text{VS} = d = -1$

$\text{Interval} = \frac{2\pi}{4} = \frac{\pi}{2}$
 $\text{Midline} = y = -1$

$0, \pi/2, \pi, 3\pi/2, 2\pi$

Handwritten notes: "right $3\pi/2$ units", "midline is $y = -1$ ", "amplitude is 3", "vertical shift down 1"

3d) $y = 4 \cos(1/2x + \pi) + 1$

$a=4$ $b=1/2$ $c=\pi$ $d=1$

$\text{Amp} = |a| = |4| = 4$
 $\text{Per} = 2\pi/b = 2\pi / (1/2) = 4\pi$
 $\text{Ps} = -c/b = -(\pi) / (1/2) = -2\pi$
 $\text{VS} = d = 1$

$\text{Interval} = \frac{4\pi}{4} = \pi$
 $\text{Midline} = y = 1$

$0, \pi, 2\pi, 3\pi, 4\pi$

Handwritten notes: "up 1 mid-line", "amplitude is 4", "vertical shift up 1", "horizontal shift right 2π "

$\pi/2$
 π
 $3\pi/2$
 2π