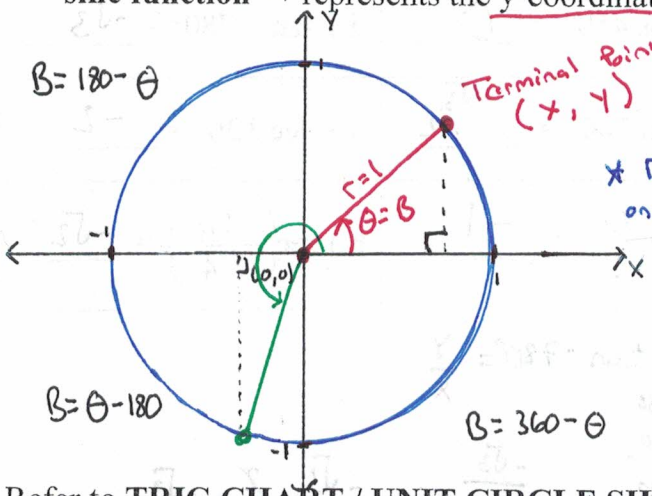
8.2 – The Unit Circle and Finding Exact Value

– **unit circle** → a circle with a radius of 1 and centered at (0 , 0) (h, k) and has equation of $x^2 + y^2 = 1$

Eqn of Circle: $(x-h)^2+(y-k)^2=r^2 \longrightarrow (x-0)^2+(y-0)^2=(1)^2 \rightarrow x^2+y^2=1$

- **reference angle** → an acute angle formed between a drawn angle θ and the x-axis.
- **terminal point** → a point (x , y) that falls on the Unit Circle. $(\underset{x}{\cos\theta}, \underset{y}{\sin\theta})$
- **cosine function** → represents the x-coordinate of the terminal point of an angle on the Unit Circle.
- **sine function** → represents the y-coordinate of the terminal point of an angle on the Unit Circle.



Rt Δ		UC		Ref Δ
$\sin\theta = \frac{O}{H}$	=	$\frac{y}{1}$	=	$\frac{y}{h}$
$\csc\theta = \frac{H}{O}$	=	$\frac{1}{y}$	=	$\frac{h}{y}$
$\cos\theta = \frac{A}{H}$	=	$\frac{x}{1}$	=	$\frac{x}{h}$
$\sec\theta = \frac{H}{A}$	=	$\frac{1}{x}$	=	$\frac{h}{x}$
$\tan\theta = \frac{O}{A}$	=	$\frac{y}{x}$	=	$\frac{y}{x}$
$\cot\theta = \frac{A}{O}$	=	$\frac{x}{y}$	=	$\frac{x}{y}$

* NO RADICALS IN DENOMINATOR ! *

Refer to **TRIG CHART / UNIT CIRCLE SHEET** to label parts of the Unit Circle:

1.) Complete the TRIG CHART → Use the 45 – 45 – right Δ and the 30 – 60 – right Δ
For quadrant angles (0° and 90°), use your calculator

2.) Label the degree measure ABOVE each pt on the Unit Circle (only use increments of 30°, 45°, 60°)

3.) Label the radian measure BELOW each pt on the Unit Circle (convert degree measure to radians)

4.) Draw diagonal lines through pairs of points that have the same reference number (angle):

a.) 30° and 210°, 150° and 330° → RED → Ref Angle = 30°

b.) 45° and 225°, 135° and 315° → BLUE → Ref Angle = 45°

c.) 60° and 240°, 120° and 300° → GREEN → Ref Angle = 60°

5.) Label the terminal point (x , y) of each degree/radian measure → (x = cos θ , y = sin θ)

6.) Write in Quadrant #'s and where trig functions are positive (ALL SENIORS TAKE CANDY)

Example 1: Using your TC/UC Sheet, answer each question.

a.) What is the reference angle for the angle of 240°? ref∠ = θ - 180
240 - 180
ref∠ = 60°

b.) What is the reference angle for the angle of $\frac{3\pi}{4}$? → 135° = QII
ref∠ = 180 - θ
180 - 135 = 45° → π/4

c.) What is the reference angle for the angle of – 750°
-750 +360 -390 +360 → -30 +360 → 330°
ref∠ = 360 - θ
360 - 330 = 30°

d.) What is the terminal point for the angle of 510°? * This is an ordered pair on the UC!
510 +360 150
$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
QII (-, +)

e.) What is the terminal point for the angle of $-\frac{9\pi}{4}$? → -405°
-405 +360 -45 +360 → 315° → 360 - 315 = 45°
QIV
$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

f.) If you are at terminal pt (0 , 1) and move 300° CCW, what angle did you stop at that is on the UC?
90 + 300 = 390
-360
30°
90 (0,1)

Steps to Find Exact Value of an Angle: Some answers contain radicals/fractions (**NO decimal answers**)

1) Find the reference angle for given angle θ – Use the "Coloring Coding key" to help determine this.

2) Use Trig Chart to look up value using reference angle found in step 1.

3) Use "Signs" Diagram of Trigonometric Functions to determine is value is positive or negative

* If finding the exact value of a quadrant angle (90°, 180°, 270°, or 360°) → use values in terminal points*

Example 2: Using your TC/UC Sheet, find the exact value. Remember – NO DECIMALS!!!!

a.) $\sin 135° =$ $\sqrt{2}/2$	b.) $\csc 210° =$ -2	c.) $\cos 450° =$ 0	d.) $\tan -780° =$ $-\sqrt{3}$
e.) $\sec 390° =$ $2\sqrt{3}/3$	f.) $\cot 180° =$ Undefined	g.) $\sin 240° =$ $-\sqrt{3}/2$	h.) $\sec 120° =$ -2
i.) $\tan\left(\frac{7\pi}{6}\right) =$ $\sqrt{3}/3$	h.) $\cos\left(\frac{5\pi}{3}\right) =$ $1/2$	j.) $\sin\left(\frac{7\pi}{2}\right) =$ -1	k.) $\sec\left(-\frac{11\pi}{4}\right) =$ $-\sqrt{2}$

2a) $\sin 135° = \frac{y}{1}$

$= \frac{\frac{\sqrt{2}}{2}}{1}$

$= \frac{\sqrt{2}}{2}$

2b) $\csc 210° = \frac{1}{y}$

$= \frac{1}{-\frac{1}{2}}$ KCF

$1 \cdot -\frac{2}{1} = -2$

2d) $\tan -780° = \frac{y}{x}$

-780

$+360$

-420

$+360$

-60

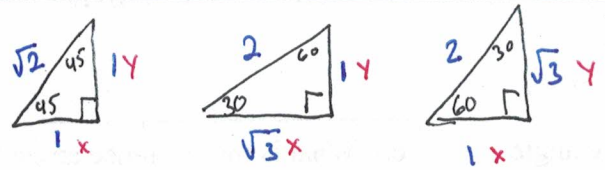
$+360$

300

$\frac{-\sqrt{3}}{2} \div \frac{1}{2} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

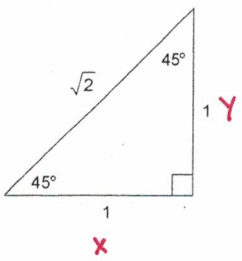
Example3: Using reference angles, find the exact value. Remember – NO DECIMALS!!!!

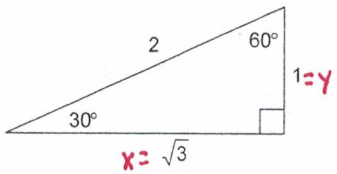
a.) $\sin 45° =$ $\sqrt{2}/2$	b.) $\cos 330° =$ $\sqrt{3}/2$	c.) $\cos 90° =$ 0	d.) $\tan -780° =$ $-\sqrt{3}$
e.) $\sin 390° =$ $1/2$	f.) $\tan 270° =$ undef	g.) $\tan 180° =$ 0	h.) $\cos 225° =$ $-\sqrt{2}/2$
i.) $\tan\left(\frac{7\pi}{6}\right) =$ $\sqrt{3}/3$	j.) $\cos\left(\frac{2\pi}{3}\right) =$ $-1/2$	k.) $\sin(-3\pi) =$ 0	l.) $\tan\left(\frac{15\pi}{4}\right) =$ -1

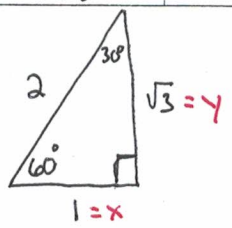


AFM – Trig Chart / Unit Circle Sheet

θ	sin θ (y)	cos θ (x)	tan θ (y / x)	csc θ (1 / y)	sec θ (1 / x)	cot θ (x / y)
0° Quadrant ∠	$\frac{0}{1} = 0$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{1}{0} =$ Undef.	$\frac{1}{1} = 1$	$\frac{1}{0} =$ Undef.
30°	$\frac{1/2}{1} = 1/2$	$\frac{\sqrt{3}/2}{1} = \sqrt{3}/2$	$\frac{1/2}{\sqrt{3}/2} = \sqrt{3}/3$	$\frac{1}{1/2} = 2$	$\frac{1}{\sqrt{3}/2} = 2\sqrt{3}/3$	$\frac{\sqrt{3}/2}{1/2} = \sqrt{3}$
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\frac{1}{\sqrt{2}/2}$ $\sqrt{2}$	$\sqrt{2}$	1
60°	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$	$\frac{1}{\sqrt{3}/2} = 2\sqrt{3}/3$	2	$\sqrt{3}/3$
90° Quadrant ∠	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{1}{0} =$ Undef	$\frac{1}{1} = 1$	$\frac{1}{0} =$ Undef	$\frac{0}{1} = 0$





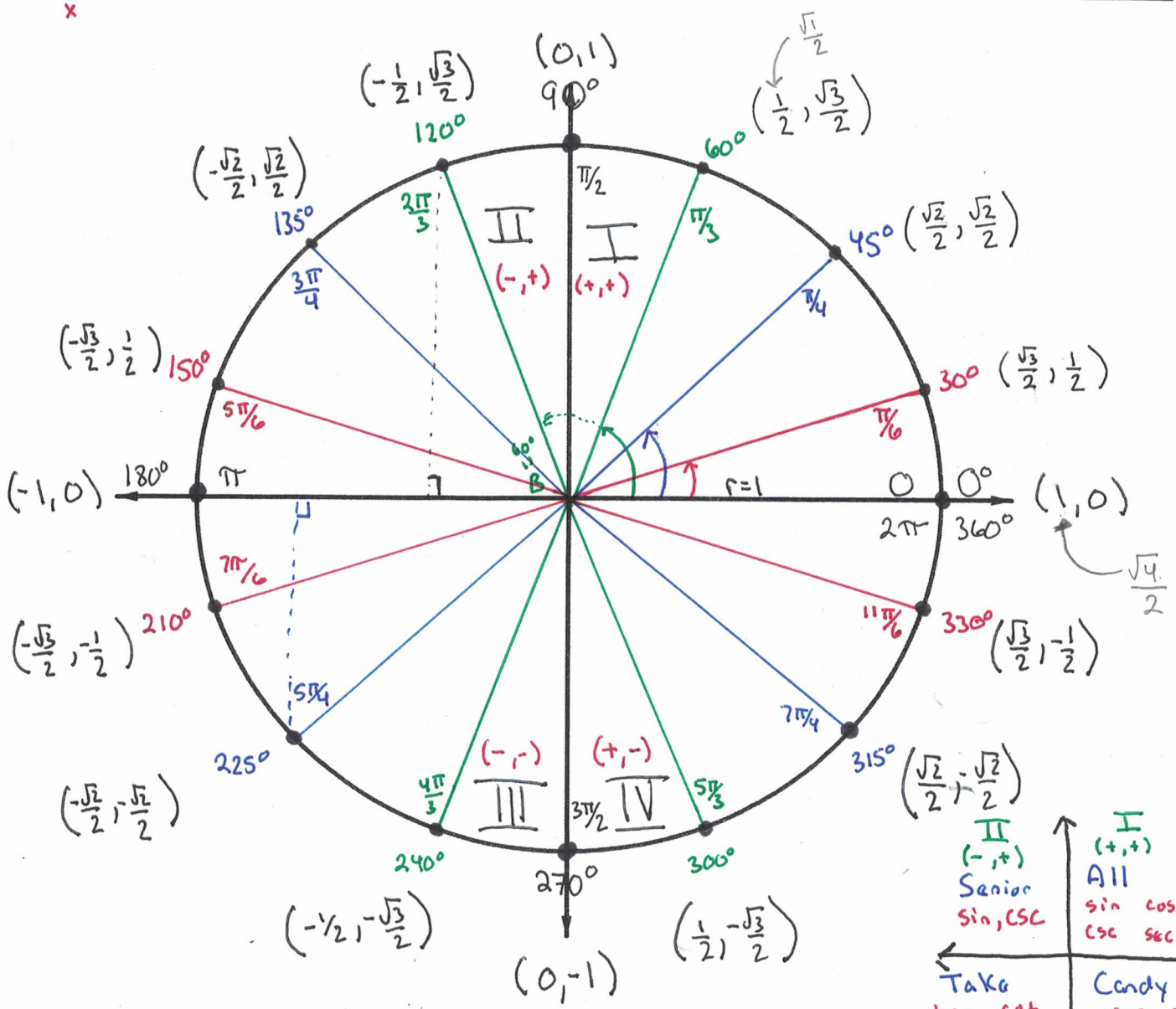


Reference Angle Color Coding Key

[ ] – reference angles = 30°

[ ] – reference angles = 45°

[ ] – reference angles = 60°



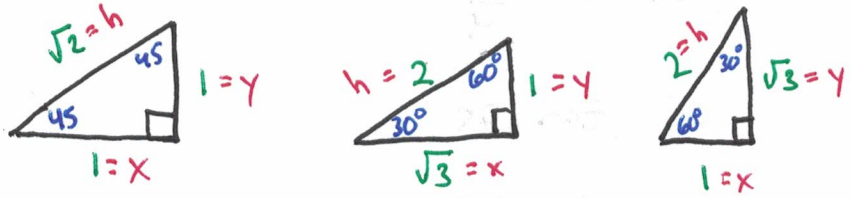
II (-,+) Senior sin, csc

I (+,+) All sin cos tan csc sec cot

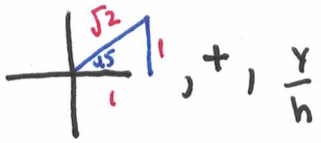
Take tan, cot (-,-) III

Candy cos, sec (+,-) IV

Ex. 3



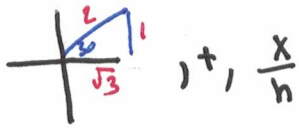
a) Sin 45°



, +, $\frac{y}{h}$

$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

b) cos 330°



, +, $\frac{x}{h}$

360 − 330 = 30°

$\frac{\sqrt{3}}{2}$

c) cos 90°

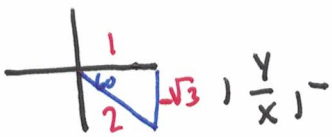
$\frac{x}{1} = \frac{0}{1} = 0$

d) tan −780

↓

tan 300

360 − 300 = 60



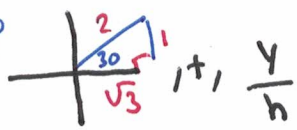
, $\frac{y}{x}$, −

$\frac{-\sqrt{3}}{1} = -\sqrt{3}$

e) sin 390°

↓

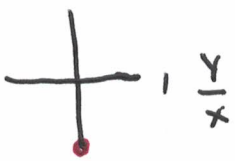
sin 30°



, +, $\frac{y}{h}$

$\frac{1}{2}$

f) tan 270°

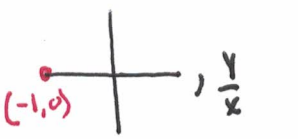


, $\frac{y}{x}$

(0, −1)

$\frac{-1}{0}$ = undefined

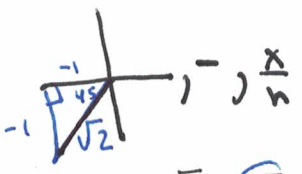
g) tan 180°



, $\frac{y}{x}$

$\frac{0}{-1} = 0$

h) cos 225

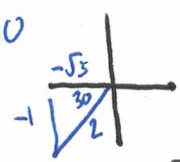


, −, $\frac{x}{h}$

$\frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

i) tan $\frac{7\pi}{6}$

tan 210

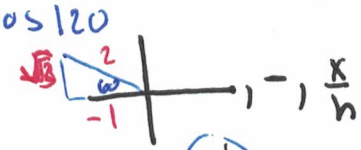


, +, $\frac{y}{x}$

$\frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

j) cos $\frac{2\pi}{3}$

cos 120



, −, $\frac{x}{h}$

$\frac{-1}{2}$

k) $\sin(-3\pi)$

$\sin \pi$

$\sin 180$

$\frac{0}{1} = 0$

l) $\tan \frac{15\pi}{4}$

$\tan 675$

$\tan 315$

$\frac{Y}{X}$ $\frac{-1}{1} = -1$