5.5 – Compounded Interest

Compounded Interest with "n" values: is an exponential function that is compounded according to a specific pay period "n", where the rate "r" is divided by "n" and the time period "t" is multiplied by "n".

$A(t) = a(1 + r)^t$

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

A = Amount (Final)
P = Principle (Initial Amount)
r = rate (% → decimal)
n = pay periods
n·t = time

Common "n" pay periods

annually: n = 1	monthly: n = 12
semiannually: n = 2	weekly: n = 52
quarterly: n = 4	daily: n = 365

Examples:

1.) John invested $500 into an account with a 3% interest rate that is compounded quarterly. How long will it take for his investment to double? $A = P(1 + \frac{r}{n})^{t}$

A = 1000 r = 3% → .03 t = ?
P = 500 n = 4

$1000 = 500(1 + \frac{.03}{4})^{4t}$

$1000 = 500(1.0075)^{(4t)}$ ← Must put in ( ) when putting into calculator!

When missing "t", put into Y= and use the TABLE to find "t".

* Looking for t *

After 24 yrs the account will double.

2.) Mike decides to invest $400 into an account that has a 6% interest rate. What is the balance in the account after 4 years if the account is being compounded monthly? $A = P(1 + \frac{r}{n})^{nt}$

A = ? n = 12
P = 400 t = 4
r = 6% → .06

$A = 400(1 + \frac{.06}{12})^{12(4)}$

$A = 400(1.005)^{48}$

* Run to the nearest cent; the hundredths position.

* Looking for A *

The balance after four yrs will be about $508.20.

3.) Desmond is investing $800 into an account with a 5% interest rate. How long will it take for the account to be $2800 if the money is compounded quarterly?

A = 2800
P = 800
r = 5% → .05
n = 4
t = ?

$2800 = 800(1 + \frac{.05}{4})^{4t}$ ← missing "t", use the calculator Y=

$2800 = 800(1.0125)^{(4t)}$ ← Put the exponent in ( ) when you enter it in the calculator.

After 26 yrs, the account will $2800.

4.) Joe wants to invest $1,500 into an account that has a 5% interest rate. Find the amount in the account after 6 years compounded quarterly.

P, r, t, n

$A = ?$
$P = 1500$
$r = 5\% \rightarrow .05$
$n = 4$
$t = 6$

$A = 1500\left(1 + \frac{.05}{4}\right)^{4(6)}$

$A = 1500\,(1.0125)^{(24)}$

The amount will be $2021.03 after 6 years.

5.) A $650 is invested into an account with a 7% interest rate. How long will it take for the investment to be $2,925 if it's compounded semiannually?

$A = 2925$
$P = 650$
$r = 7\% \rightarrow .07$
$n = 2$
$t = ?$

$2925 = 650\left(1 + \frac{.07}{2}\right)^{2t}$ ← missing "t" use Y=

$2925 = 650(1.035)^{(2t)}$ Put into Y=

It will take about 22 yrs for the account to be $2925.

6.) A $2,225 is invested in an account for 5 years. What is the interest rate if the investment triples and is compounded monthly? $A = P\left(1 + \frac{r}{n}\right)^{nt}$

P, t, n, x3

2225×3

$A = 6675$
$P = 2225$
$r = ?$
$n = 12$
$t = 5$

$6675 = 2225\left(1 + \frac{r}{12}\right)^{12(5)}$

$\frac{6675}{2225} = \frac{2225}{2225}\left(1 + \frac{r}{12}\right)^{60}$

* Solve eqn for "r". do not distribute!

Means $2225 \cdot (\cdots)$, how do you undo multiplication?

$3 = \left(1 + \frac{r}{12}\right)^{60}$ ← To undo the power of 60, take the 60th root.

In Calculator
60 MATH #5 3 Enter
60 $\sqrt[x]{}$ 3

$\sqrt[60]{3} = \sqrt[60]{\left(1 + \frac{r}{12}\right)^{60}}$

$1.018478864 = 1 + \frac{r}{12}$ (−1 from each side)

$(12)(.018478864) = \frac{r}{12}(12)$

$r = .2217463732$ ← decimal need a %

The rate is 22.2%.