Math 2

order matters!

Unit 7.2 – Permutations and Combinations

– **permutation** → a group of objects or people that are arranged in a CERTAIN order

- Words that indicate a permutation – arrange / arrangement or placement (1st, 2nd, 3rd place; President, Vice Pres.)
  also the arrangement of letters of a word.
- Simple Example – A softball team's manager needs to fill out a card before the game for the team's lineup at bat. The manager has 7 possible players in mind for the top 4 spots in the lineup. How many arrangements can the manager write the lineup?
  Use the Fundamental Counting Principle → $7 \times 6 \times 5 \times 4 = 840$ different line ups.

Computing a Permutation With a Formula (with no repetitions) →

The number of permutations of n distinct objects taken r at a time is given by P (n , r) = $\frac{n!}{(n-r)!}$ ← Permutation

↳ total # (Big) ↳ taken/selected (small)

Computing a Permutation With a Calculator (with no repetitions) →

Use notation $_nP_r$ and follow these steps:
1.) type in "n" first (always will be the bigger # of the two)
2.) MATH , scroll to PRB , select # 2 (nPr)
3.) type in "r" (always will be the smaller # of the two)

P(7, 4)

[7] [MATH] → PRB #2: nPr [4] [Enter] = 840 ways

Computing a Permutation With a Formula (with repetitions) →

The number of permutations of n objects of which p are alike and q are alike is given by $\frac{n!}{(p!\,q!\ldots)}$ ← total; repeaters

9 total → ISOSCELES 2 E's, 3 S's

Example 1: The following examples are permutations –

a.) There are 10 finalists (n) in a figure skating competition.
How many ways can gold, silver, and bronze medals are awarded? r = 3
↳ indicates order (Permutation)

Using the Formula	Using the Calculator
$P(10,3) \rightarrow \frac{10!}{(10-3)!} \Rightarrow \frac{10!}{7!} = 720$ ways $\frac{10 \times 9 \times 8 \times \cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}{\cancel{7} \times \cancel{6} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{1}}$	$P(10,3) \rightarrow {}_{10}P_3$ 720 ways

Permutation

b.) placing an algebra book, a geometry book, a chemistry book, an English book, and a health book on a shelf
total of 5 books = n
using/taking all 5 books = r
$P(5,5) \rightarrow {}_5P_5 = 120$ ways to arrange

c.) The manager of a four-screen movie theater is deciding which of 12 available movies (n) to show. The screens are in rooms with different seating capacities. How many ways can he show four different movies (r) on the screens? $P(12,4) \rightarrow {}_{12}P_4 = 11{,}880$ ways

d.) How many different ways can the letters of the following words be arranged? – Permutation!

1.) MISSISSIPPI – 11 total
$\frac{n!}{(p!\,q!\,r!)}$
4 I's, 4 S's, 2 P's (repeaters)
$\frac{11!}{(4!\,4!\,2!)} = 34{,}650$ arrangements

2.) PARALLEL – 8 total
2 A's, 3 L's (repeating letters)
$\frac{8!}{(2!\,3!)} = 3{,}360$ arrangements

3.) COMPANY
7 total
No repeating letters!
$P(7,7) \rightarrow {}_7P_7 = 5040$ ways

Math 2

order does not matter

– **combination** → a group of objects or people that are NOT arranged in certain order

- Words that indicate a combination – select / selecting or choose / choosing
- Simple Example – The Coolidge Family is ordering pizza for dinner. They must choose 2 toppings out of a list of 6.

Computing a Combination With a Formula →

The number of combinations of n distinct objects taken r at a time is given by C (n , r) = $\frac{n!}{((n-r)!\,r!)}$

Total Big · Taken Small

Computing a Combination With a Calculator →

Use notation $_nC_r$ and follow these steps: 1.) type in "n" first (always will be the bigger # of the two)
4.) MATH , scroll to PRB , select # 3 (nCr)
5.) type in "r" (always will be the smaller # of the two)

$C(6,2) \rightarrow {}_6C_2 = 15$ combos

Example 2: The following examples are combinations –

a.) A group of seven students working on a project needs to choose two from their group to present the group's report to the class. How many ways can they choose the two students?

Combination

Using the Formula	Using the Calculator
$C(7,2)$ $\frac{7!}{((7-2)!\,2!)} \Rightarrow \frac{7!}{(5!\,2!)} = 21$ ways	$C(7,2) \rightarrow {}_7C_2 = 21$ ways

$\frac{7\times \cancel{6}^{3}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{(\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})(\cancel{2}\times 1)} = \frac{7\times 3}{1} = 21$

b.) selecting three of fifteen flavors of ice cream at the grocery store

combo

$C(15,3) \rightarrow {}_{15}C_3 = 455$ ways

c.) selecting nine books to check out of the library from a reading list of twelve

Combo

$C(12,9) \rightarrow {}_{12}C_9 = 220$ ways

d.) The principal at Cobb County High School wants to start a mentoring group. He needs to narrow his choice of students to be mentored to six from a group of nine. How many ways can a group of six to be selected?

combo

$C(9,6) \rightarrow {}_9C_6 = 84$ ways

Example 3: Determine if the situations below are PERMUTATIONS or COMBINATIONS.
Remember: Permutations → Order matters ; Combinations → Order doesn't matter

a.)	b.)	c.)	d.)
a.) A chemistry teacher divides his class into eight groups. Each group submits one drawing of the molecular structure of water. He will select four of the drawings to display. In how many different ways can he select the drawings? NO ORDER	b.) You will draw winners from a total of 25 tickets in a raffle. The first ticket wins \$100. The second ticket wins \$50. The third ticket wins \$10. In how many different ways can you draw the three winning tickets? ORDER!	c.) How many different 5 letter codes can you make from the letters in the word CUSTOMER? 8 total Arrangement of letters! No Repeaters!	d.) You have 20 songs on your iPhone. You have time to listen to three of the songs. In how many ways can you choose the three songs?
Combination $C(8,4)$	Permutation $P(25,3)$	Permutation $P(8,5)$	Combination $C(20,3)$

Combo (a.) · Combo (d.)

$_8C_4 = 70$ ways

$_{25}P_3 = 13{,}800$ ways

$_8P_5 = 6720$ codes

$_{20}C_3 = 1140$ ways