

# 8.1 (Part I) – Angles and Angle Measure

General Angle in Standard Position	Various Types of Common Angles		
	Positive Angles	Negative Angles	Quadrant Angles
<p><b>angle</b> → is a rotation of a <b>RAY</b> about a <b>FIXED</b> point.</p> <p>Terminal Side (Rotates)</p> <p>Initial Side (Stationary)</p> <p>Vertex (0,0)</p>	<p>rotation is <b>CCW</b> (counter-clockwise)</p> <p>arrow is <b>ON TOP</b> of the initial side!</p>	<p>rotation is <b>CW</b> (clockwise)</p> <p>arrow is <b>UNDER</b> the initial side.</p>	<p>Terminal side falls on</p> <p>a.) x-axis like <math>0^\circ, \pm 180^\circ, \pm 360^\circ</math></p> <p>b.) y-axis like <math>\pm 90^\circ, \pm 270^\circ</math></p>

Angle Measurement # 1 – Degrees (with $^\circ$ )	Angle Measurement # 2 – Radians (with $\pi$ )
<p>To convert from degrees (<math>^\circ</math>) to radians (<math>\pi</math>) →</p> <p><math>(\text{degrees})^\circ \cdot \frac{\pi}{180} = \pi \text{ rads}</math></p> <p>Ex: Convert given degree measure to radians:</p> <p>a.) <math>45^\circ \rightarrow 45^\circ \cdot \frac{\pi}{180} \rightarrow \frac{45}{180} = \frac{1}{4} \rightarrow \frac{\pi}{4}</math></p> <p>↳ is <math>\frac{\pi}{4}</math> or .7853981634 rads</p> <p>b.) <math>300^\circ \rightarrow 300^\circ \cdot \frac{\pi}{180} \rightarrow \frac{300}{180} = \frac{5}{3} \rightarrow \frac{5\pi}{3}</math></p> <p>↳ 5.235987756 rads</p>	<p>To convert from radians (<math>\pi</math>) to degrees (<math>^\circ</math>) →</p> <p><math>\text{RAD } \pi \cdot \frac{180}{\pi} = \text{degrees}</math></p> <p>Ex: Convert given radian measure to degrees:</p> <p>a.) <math>\frac{\pi}{3} \rightarrow \frac{\pi}{3} \cdot \frac{180}{\pi} = 180/3 = 60^\circ</math></p> <p>b.) <math>\frac{5\pi}{6} \rightarrow \frac{5\pi}{6} \cdot \frac{180}{\pi} = \frac{900}{6} = 150^\circ</math></p> <p>↳ 2.617993878 rads · 180 = 150°</p>

\* If you start in radians, you must end in radians!

**Example 1:** Draw each angle in standard position. Draw the arrow of angle's direction.

a.) $\theta = 48^\circ$	b.) $\theta = -212^\circ$	c.) $\theta = 270^\circ$	d.) $\theta = \frac{4\pi}{3}$	e.) $\theta = -\frac{\pi}{6}$	f.) $\theta = -\pi$

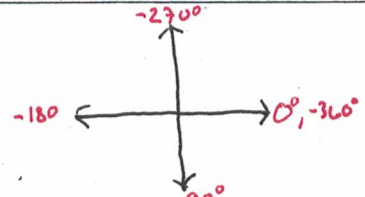
– **coterminal angles** → angles that **SHARE** a terminal side (end in the same place)

- To find a **POSITIVE** coterminal → **ADD  $360^\circ$**  (if in deg) or **ADD  $2\pi$**  (if in rads)
- To find a **NEGATIVE** coterminal → **SUBTRACT  $360^\circ$**  (if in deg) or **SUBTRACT  $2\pi$**  (if in rads)
- Coterminal angles can contain **multiple rotations** (I call these “swirlies”)

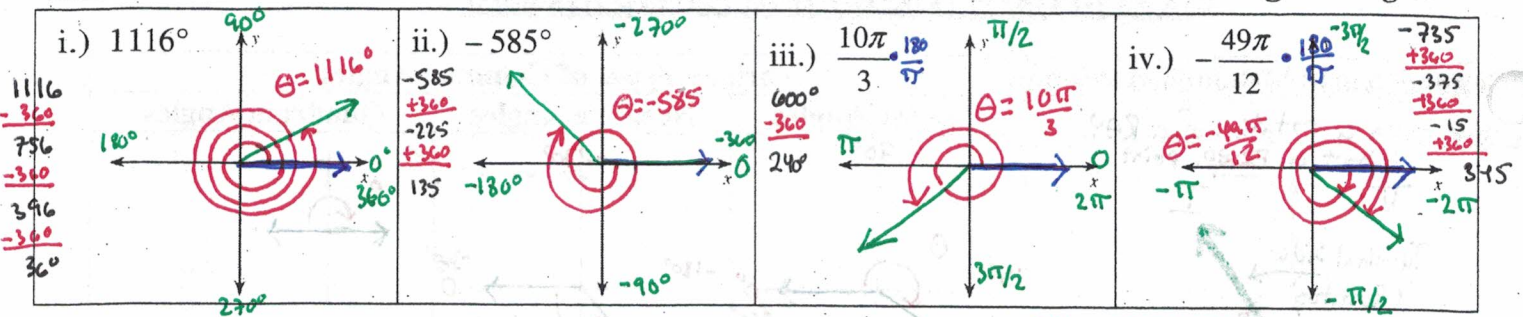
**Example 2a:** Find a positive and negative coterminal angle for the given angle  $\theta$ .

<p>i.) <math>\theta = 60^\circ \rightarrow</math></p> <p>positive coterminal angle = <math>60^\circ + 360^\circ = 420^\circ</math></p> <p>negative coterminal angle = <math>60^\circ - 360^\circ = -300^\circ</math></p>	<p>ii.) <math>\theta = \frac{7\pi}{6} \rightarrow</math></p> <p>positive coterminal angle = <math>\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}</math></p> <p>negative coterminal angle = <math>\frac{7\pi}{6} - 2\pi = -\frac{5\pi}{6}</math></p>
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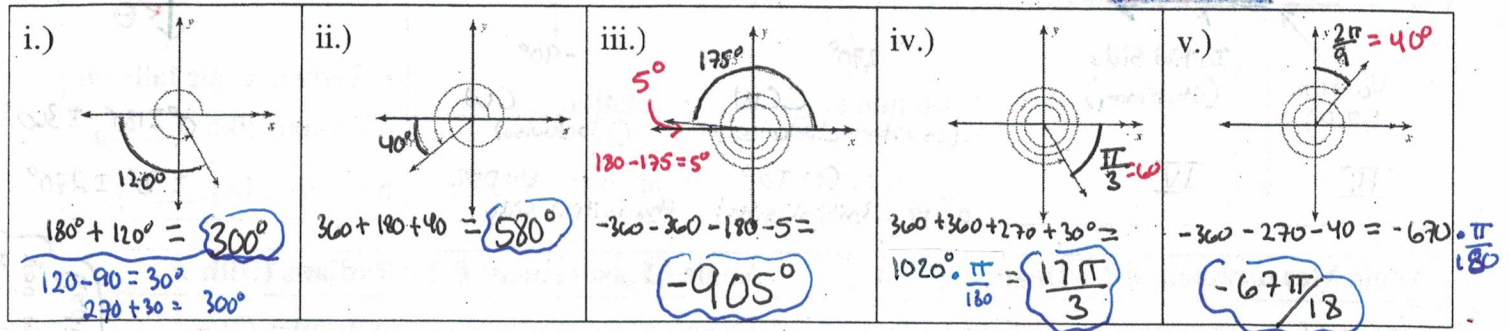
Degrees	Radian
$0^\circ$	$0$
$90^\circ$	$\frac{\pi}{2}$
$180^\circ$	$\pi$
$270^\circ$	$\frac{3\pi}{2}$
$360^\circ$	$2\pi$



**Example 2b:** Find and draw an angle between  $0^\circ$  and  $360^\circ$  that is coterminal with the given angle.



**Example 2c:** Find the measure of each angle using the given picture. **Keep units consistent.**



– **reference angle** → an angle  $B$  associated with angle  $\theta$  where  $B$  is an acute angle formed by the terminal side of angle  $\theta$  and the  $x$ -axis.

Angle $\theta$ Falls in QI	Angle $\theta$ Falls in QII	Angle $\theta$ Falls in QIII	Angle $\theta$ Falls in QIV
Formula: $B = \theta$	Formula: $B = 180 - \theta$	Formula: $B = \theta - 180$	Formula: $B = 360 - \theta$

**Example 3:** Find the reference angle  $B$  given angle  $\theta$ . Make sure angle  $\theta$  is between  $0^\circ$  and  $360^\circ$

angle $\theta$	Quadrant $\theta$ Lies	Work to find angle $B$	reference angle $B$
a.) $120^\circ$	II	$B = 180 - \theta \rightarrow 180 - 120$	$B = 60^\circ$
b.) $53^\circ$	I	~~~~~	$B = 53^\circ$
c.) $948^\circ$	III	$B = \theta - 180 \rightarrow 228 - 180$	$B = 48^\circ$
d.) $-765^\circ$	IV	$B = 360 - \theta \rightarrow 360 - 315$	$B = 45^\circ$

Acute Angles

c)  $948^\circ$   
 $-360$   
 $588$   
 $-360$   
 $228^\circ$

d)  $-765^\circ$   
 $+360$   
 $-405$   
 $+360$   
 $-45$   
 $+360$   
 $315^\circ$