Math 2

Unit 7.1 – The Fundamental Counting Principle

Outcome ~~outcome~~ → the result of a single trial

Ex: The trial of flipping a coin once has two outcomes either heads or tails

- **sample space** → the {SET} of ALL possible outcomes ie.. → coin: {head, tail} → die: {1, 2, 3, 4, 5, 6}

- **event** → one or more outcomes of a trial.

- independent events – the outcome of one event DOES NOT affect the outcome of another event

  Ex: tossing a coin or rolling a die

- dependent events – the outcome of one event DOES affect the outcome of another event

  Ex: taking a piece of candy from a jar and then taking a second piece without replacing the first

There are two ways to determine the possible outcomes for either events →

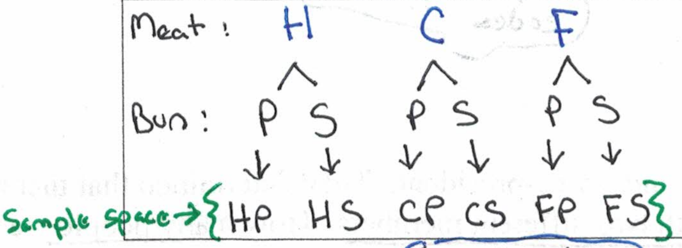
- Visually → create a tree diagram or a table which is particularly useful for independent events
- Mathematically → use the Fundamental Counting Principle which is useful for several multiple choices of independent events or various dependent events

Fundamental Counting Principle →

IF *event M can occur in m ways and is followed by event N that can occur in n ways, then event M followed by event N can occur in* $m \cdot n$ ways

Example 1: The following examples are independent events –

a.) A sandwich cart offers customers a choice of hamburger, chicken, or fish on either a plain or sesame seed bun. How many different combinations of meat and a bun are possible?

Visual Method	Mathematical Method
	3 ways to choose meat 2 ways to choose bun $3 \cdot 2 = 6$ combinations

(6 combinations)

b.) Kim won a contest on a radio station. The prize was a restaurant gift certificate to one of the city's three best restaurants and tickets to the following sporting events: football, baseball, basketball, or hockey. How many different ways can she select her prize?

3 ways to choose a restaurant
4 ways to choose a sporting event

$3 \cdot 4 = 12$ different ways

c.) Many answering machines allow owners to call home and get their messages by entering a 3-digit code. How many codes are possible?

* digits 0–9 *

1st digit : 10 choices
2nd digit : 10 choices
3rd digit : 10 choices

$10 \cdot 10 \cdot 10 = 1000$

1000 possible codes

Example 2: The following examples are dependent events –

a.) Charlene wants to take 6 different classes next year. Assuming that each class is offered each period, how many different schedules could she have?

Visual Method

Periods	1st	2nd	3rd	4th	5th	6th
# of Class choices	6	5	4	3	2	1

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720 \text{ possible schedules}}$

Mathematical Method

$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = \boxed{720 \text{ possible schedules}}$

$6!$ ← Factorial

it is read as "six factorial"

Calculator

$\boxed{6}$ $\boxed{\text{MATH}}$ → PROB → 4: ! $\boxed{\text{Enter}}$ = 720

(Probability)

b.) Complete the given problem –
Assume that all the books are different:

$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

or

$9! = \boxed{362{,}880 \text{ book arrangements}}$

c.) A computer's 6 character password can be formed if the first two characters are letters and the remaining characters are digits where neither character can't be repeated. How many possible passwords could there be?

Letters: 26

digits: 0–9 → 10

characters	1st	2nd	3rd	4th	5th	6th
Choices	26	25	10	9	8	7
	letters		digits			

No Repeat

$26 \cdot 25 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = \boxed{3{,}276{,}000 \text{ possible passwords.}}$

d.) How many different 5-digit codes are possible (referring to a key pad) if the first digit can not be 0 and rest of the digits after the first can be used more than once?

Characters	1st	2nd	3rd	4th	5th
Choices	9	10	10	10	10
	can't be 0!	can be zero and repeat!			

$9 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = \boxed{90{,}000 \text{ possible codes}}$

Example 3: Critical Thinking Problem

The members of the Math Club need to elect a president and a vice-president. They determined that there are a total of 272 ways that they can fill the positions with two different members. How many people are in the Math Club?

X = # of members for 1st position (President)

X−1 = # of members for 2nd position (Vice President)

$x(x-1) = 272$ total outcomes.

$x^2 - x = 272$

Quadratic → $x^2 - x - 272 = 0$

$a = 1 \quad b = -1 \quad c = -272$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-272)}}{2(1)} \rightarrow x = \frac{1 \pm \sqrt{1089}}{2}$$

$$x = \frac{1+33}{2} \qquad x = \frac{1-33}{2}$$

$$x = 17 \qquad x = \cancel{-16}$$

$\boxed{17 \text{ members}}$