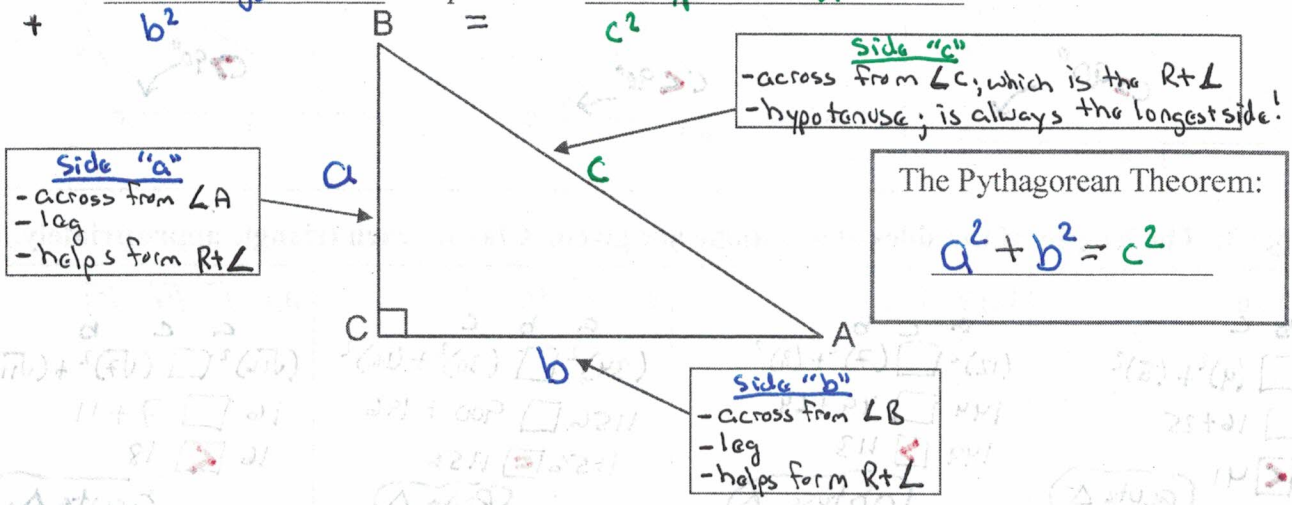
Unit 6.1 – The Pythagorean Theorem and Its Converse

The Pythagorean Theorem

This theorem states if a triangle is a right triangle, then the sum of a $(\text{leg})^2$ a^2 and the $(\text{other leg})^2$ b^2 is equal to the $(\text{hypotenuse})^2$ c^2.



Side "a"
- across from ∠A
- leg
- helps form Rt∠

Side "b"
- across from ∠B
- leg
- helps form Rt∠

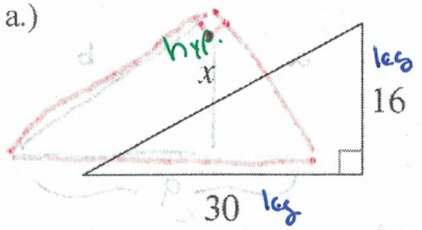
Side "c"
- across from ∠C, which is the Rt∠
- hypotenuse; is always the longest side!

The Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

Example 1: Use the Pythagorean Theorem to find the missing length x.

a.)



$(30)^2 + (16)^2 = (x)^2$

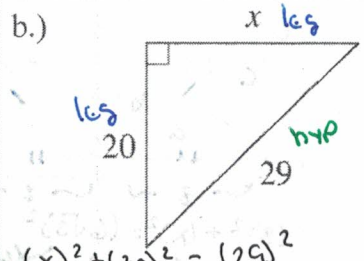
$900 + 256 = x^2$

$1156 = x^2$

$\sqrt{1156} \quad \sqrt{x^2}$

$x = 34$

b.)



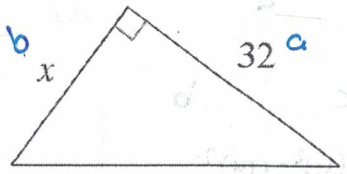
$(x)^2 + (20)^2 = (29)^2$

$x^2 + 400 = 841$

$x^2 = 441$

$x = 21$

c.)



$(32)^2 + (x)^2 = (40)^2$

$1024 + x^2 = 1600$

$x^2 = 576$

$x = 24$

The Pythagorean Triples

- Three numbers that ALWAYS work in the Pythagorean Theorem!.
- Always plug in the LARGEST number for c → why? c is alway hypotenuse; which is largest side.
- Common Pythagorean Triples: 3, 4, 5 — 5, 12, 13 — 8, 15, 17 — 7, 24, 25

Multiply any of these common triples by the same positive whole number, it will produce more sets!

Example 2: Determine if each set of numbers form a Pythagorean Triple.

a.) 7, 8, 9 (a b c)

$(7)^2 + (8)^2 = (9)^2$

$49 + 64 = 81$

$113 \neq 81$ NO

b.) 38, 21, 28 (c a b)

$(21)^2 + (28)^2 = (38)^2$

$441 + 784 = 1444$

$1225 \neq 1444$ NO

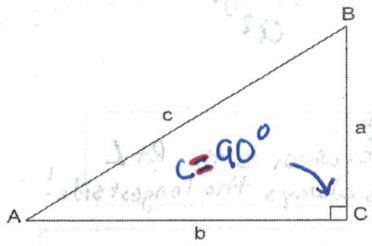
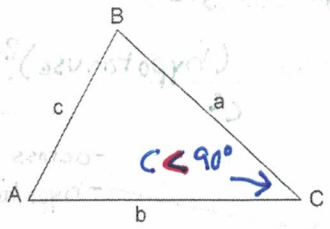
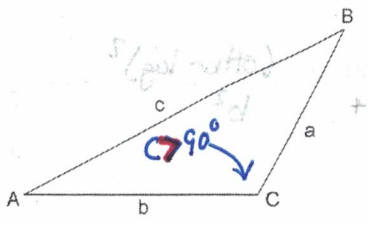
c.) 15, 25, 20 (a c b)

$(15)^2 + (20)^2 = (25)^2$

$225 + 400 = 625$

$625 = 625$ ✓ YES

The Converse of the Pythagorean Theorem

If $c^2 = a^2 + b^2$, then Δ will be a(n) Right Δ.	If $c^2 < a^2 + b^2$, then Δ will be a(n) acute Δ.	If $c^2 > a^2 + b^2$, then Δ will be a(n) obtuse Δ.
		

Example 3: The lengths of the sides of a triangle are given. Classify each triangle appropriately.

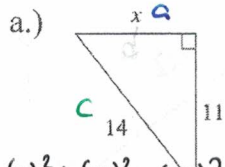
a.) 4, 5, 6 (a b c)
$(6)^2 \square (4)^2 + (5)^2$
$36 \square 16 + 25$
$36 < 41$
acute Δ

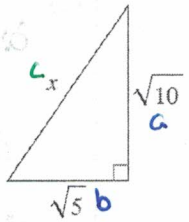
b.) 7, 12, 8 (a c b)
$(12)^2 \square (7)^2 + (8)^2$
$144 \square 49 + 64$
$144 > 113$
obtuse Δ

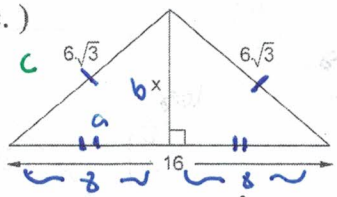
c.) 30, 16, 34 (a b c)
$(34)^2 \square (30)^2 + (16)^2$
$1156 \square 900 + 256$
$1156 = 1156$
Right Δ

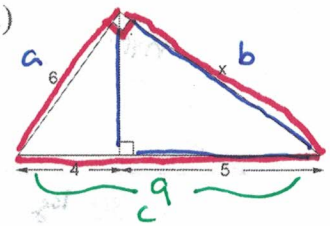
d.) $\sqrt{7}, \sqrt{16}, \sqrt{11}$ (a c b)
$(\sqrt{16})^2 \square (\sqrt{7})^2 + (\sqrt{11})^2$
$16 \square 7 + 11$
$16 < 18$
acute Δ

Example 4: Find the missing side x. Leave answers in radical form. Simplified Radical Form!

a.) 
$(x)^2 + (11)^2 = (14)^2$
$x^2 + 121 = 196$
$x^2 = 75$
$\sqrt{x^2} = \sqrt{75}$
$x = 5\sqrt{3}$
75 = 3 · 25 = 5 · 5

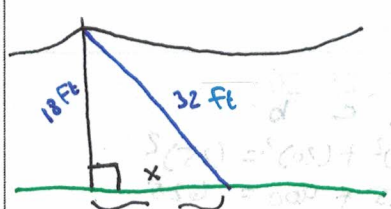
b.) 
$(\sqrt{10})^2 + (\sqrt{5})^2 = (x)^2$
$10 + 5 = x^2$
$15 = x^2$
$x = \sqrt{15}$

c.) 
$(8)^2 + (x)^2 = (6\sqrt{3})^2$
$64 + x^2 = 108$
$(6)^2 \cdot (\sqrt{3})^2$
$36 \cdot 3$
$x^2 = 44$
$x = 2\sqrt{11}$
44 = 2 · 22 = 2 · 11

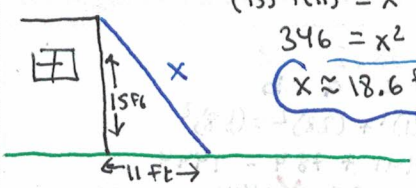
d.) 
$(6)^2 + (x)^2 = (9)^2$
$36 + x^2 = 81$
$x^2 = 45$
$x = 3\sqrt{5}$
45 = 3 · 15 = 3 · 15

Example 5: For the following – a.) Draw a picture representing each word problem.
b.) Solve for what the problem is asking for. Round to tenth place.

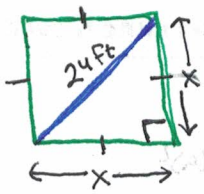
a.) A telephone support cable attaches to the pole 18 feet high. If the cable is 32 feet long, how far from the bottom of the pole does the cable attach to the ground?


$(x)^2 + (18)^2 = (32)^2$
$x^2 = 700$
$x \approx 26.5$ ft

b.) Tara leaned a ladder against her house. The bottom of the ladder is 11 feet from the house and the top of the ladder is 15 feet above the ground? How long is the ladder?


$(15)^2 + (11)^2 = x^2$
$346 = x^2$
$x \approx 18.6$ ft

c.) A walkway forms one diagonal of a square playground. The walkway is 24 meters long. How long are the sides of the playground?


$(x)^2 + (x)^2 = (24)^2$
$x^2 + x^2 = 576$
$2x^2 = 576$
$x^2 = 288$
$x \approx 17$ m